Fiscal Compact and Debt Consolidation Dynamics*

Luca Brugnolini & Luisa Corrado

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Abstract

We analyse the macroeconomic effects of a debt consolidation policy in the Euro Area mimicking the *Fiscal Compact Rule* (FCR). The rule requires the signatory states to target a debt-to-GDP ratio below 60%. Within the context of Dynamic Stochastic General Equilibrium models (DSGE), we augment a fully micro-founded New-Keynesian model with a parametric linear debt consolidation rule, and we analyse the effects on the main macroeconomic aggregates. To fully understand its implications on the economy, we study different debt consolidation scenarios, allowing the excess debt to be re-absorbed with different timings. We show that including a debt consolidation rule can exacerbate the effects of the shocks in the economy by imposing a constraint on the public debt process. Secondly, we note that the effect of loosening or tightening the rule in response to a shock is heterogeneous. Shocks hitting nominal variables (monetary policy shock) are not particularly sensitive. On the contrary, we prove that the same change has a more pronounced effect in case of shock hitting real variables (productivity and public spending shocks). Finally, we show that the macroeconomic framework worsens as a function of the rigidity of the debt consolidation rule. As a limiting case, we show that the effects on output, employment, real wages, inflation, and interest rates are sizable.

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Corresponding authors: ***Luca Brugnolini*** *brugnolinil@centralbankmalta.org, senior economist, Central Bank of Malta's Research Department.* ***Luisa Corrado*** *lc242@econ.cam.ac.uk full professor, University of Rome Tor Vergata, and University of Cambridge.*

1 Introduction

Since March 2012, all but two member states of the European Union (EU) have signed the *Treaty of Stability, Coordination and Governance* (TSCG)[1]. The result has been a tightening in the provisions of the *Stability and Growth Pact* (SGP) which requires member states not to exceed a 3% threshold of the primary deficit-to-GDP ratio each year. The tightening is reflected in the adoption of the *Fiscal Compact Rule* (FCR) that requires the signatory states to follow a structural balanced budget rule and to reduce the debt-to-GDP ratio to 60% within twenty years. Against this politico-economic background, we take a positive stand and develop a framework to analyse the impact of a debt consolidation policy mimicking the EU fiscal compact rule within a structural New-Keynesian (NK) model. Our study is motivated by the large interest triggered in Europe by the sovereign debt crisis and by the intense debate around the effects of the debt consolidation policy. In particular, a crucial issue is whether implementing such a rule in a context of negative economic shocks is still desirable. From a normative point of view, the fiscal compact rule only imposes additional constraints on the government and assuming a benevolent planner it is always suboptimal. Moreover, one of the main results of the fiscal policy literature is the optimality of the countercyclical debt policy (Barro, 1979; Lucas and Stokey, 1983; Aiyagari et al., 2002), which prescribes that in case of a negative shock ("bad times") the government should run a deficit to finance the provision of public goods, while in "good times" it should accumulate a surplus. The fiscal compact rule prescribes exactly the opposite. Therefore, in bad times, the government should run a surplus, as the debt-to-GDP ratio would deteriorate.

From a positive perspective, there is still an open debate on the optimality of a debt consolidation process. In fact, evidence provided by Giavazzi and Pagano (1990) and the European Commission (2003) suggests that about half of the EU countries starting a fiscal consolidation route between 1970 and 2000 have been experiencing immediate output growth. This finding is stark in contrast with the standard Keynesian theory (Keynes, 1936), which states that increasing public spending also increases the income level. From a positive point of view, understanding the effects of debt consolidation policies represents a crucial challenge for the fiscal policy literature, and in this paper, we take a step in this direction. In particular, we build on the New-Keynesian literature and analyse the case of the EU fiscal compact rule. In New-Keynesian models, the planner's role is limited to decide the consumption of public good for each expenditure level. Instead, we include a debt consolidation process directly into the government spending equation, and we analyse the response of the endogenous variables in the model to a battery of shocks by varying the degree of restriction imposed by the debt consolidation rule. In particular, we are interested in understanding the inter-

[1] The United Kingdom and The Czech Republic. Also, Croatia joined the EU (July 2013) without signing the treaty.

action between macroeconomic variables under the rule, and a one-time negative shock hitting the economy. The novelty of the paper is twofold: first, we develop a framework to study linear debt consolidation rule in a New Keynesian model. In fact, in this class of models, public spending is usually treated as an exogenous random variable distributed as an autoregressive process. We extend this rule by adding a deterministic component to the standard stochastic part. Therefore, the public spending rule is made by two separate elements: the first is random and represents the standard government spending shock; the second is deterministic and embeds the consolidation policy rule. The role of the deterministic component is to constrain the government to implement a debt consolidation policy, and to rule-out deficit spending imposing cuts whenever the government deviates from the target. The choice of a New-Keynesian model is not by coincidence. First, this class of models has demonstrated the ability to generate fluctuations similar to the data and replicating the main business cycle features. Secondly, and most importantly for our analysis, Smets and Wouters (2003, 2007) have shown that in a NK model with Calvo price setting, a proxy for potential output can be readily available by setting to zero the degree of price stickiness. Given that the fiscal compact rule is defined with respect to potential output, we can exploit this feature to retrieve directly into the NK model a potential output proxy, and defining the debt consolidation rule as a function of it.

As a second contribution, we calibrate the model on the Euro Area and study the effects of the rule on the main macroeconomic aggregates. Also, we parametrize the model in a way that the rule fully depends on a parameter α_g, which governs the debt consolidation dynamics. By varying the parameter, we are able to vary the tightness of the rule and to study the effects of an entire spectrum of different debt consolidation policies.

The main findings of the paper are the following; first, we show that the debt consolidation rule acts as a propagator mechanism in the impulse response functions by imposing a restriction on their dynamics. Secondly, we show that in case of a negative shock hitting nominal variables (monetary policy shock) the effects are mild, while in the case of a negative shock hitting real variables (technology and public spending shock), the macroeconomic framework worsens as a function of the rigidity of the debt consolidation rule. As a limiting case, we show that the effects on output, employment, real wages, inflation, and interest rates are sizable.

Finally, we propose an alghorithm to sistematically address sensitivity issues in the calibration of New Keynesian models, and we show that the main results of the paper remain unchanged.

The reminder of the paper is structured as follows. Section 2 motivates the analysis, and shows the sets of consolidation rules covered in the article. Section 3 develops the micro-founded model, and in section 4 we higlights the model calibration. Section 5 shows the results. Finally, section 6 concludes.

2 Motivation

Figure 1 shows the distribution of the debt-to-GDP ratio for the 28 EU countries in 2012. The black horizontal line represents the maximum level of debt attainable after the ratification of the treaty as well as the target that countries have to reach in twenty years. Analysing the chart, it is easy to see that countries are highly heterogeneous with respect to their debt levels. Having a different stock of debt leads to different constraints in terms of public spending. Signatory countries that have a debt-to-GDP ratio above the horizontal line (red bins) are required to follow a debt consolidation path according to the FCR, while countries below the horizontal line (blue bins) can keep their budget balanced (or in case running a minor deficit). For this reason, the FCR has different implications for different countries. According to the rule, *high debt countries*, i.e., those having a debt-to-GDP level above the horizontal line, have to reduce the amount of debt below the horizontal line by spreading it linearly in the following years. To examine the implication of this particular consolidation path, in equation (1) we modeled the FCR as a linear cut depending on a parameter $\alpha \in [0,1]$, which reflects both the amount and the duration of the reduction.

$$\frac{b_{t+1}}{y_{t+1}} \leq \frac{b_t}{y_t} + \alpha \left(\frac{b^*}{y^*} - \frac{b_0}{y_0} \right) \tag{1}$$

Where b_t/y_t is debt-to-GDP, b^*/y^* is the target debt-to-GDP and b_0/y_0 is the initial stock of debt-to-GDP when the rule enters into force. According to this formulation, α is the inverse of the number of years needed to implement the debt reduction to the target level ($\alpha = 1/\delta, \delta \in \mathbb{R}^+$). As $\delta \longrightarrow \infty$, $\alpha \longrightarrow 0$, and the rule implies a balanced budget rule (BBR). While, as $\delta \longrightarrow 1$, also $\alpha \longrightarrow 1$, meaning the target must be reached in one period. Also, the term in brackets implies that when the target is smaller than the initial level of debt ($b^*/y^* < b_0/y_0$), the rule implies a debt reduction. While, in the opposite situation ($b^*/y^* > b_0/y_0$) it allows for deficit spending[2]. Applying equation (1), we compute the expected consolidation path for the 25 EU signatory states, and we show in table 1 their total debt reduction and the annual surplus required every year to remain on that path. Mimicking the FCR we set $b^*/y^* = 60\%$, $\alpha = 0.05$, and b_0/y_0 to the 2012 gross central government debt for each country[3]. Countries marked in red refer to those that have to implement some form of debt consolidation, while blue marked countries are not constrained by the rule . High debt countries

[2]To rule-out deficit spending a piece-wise linear rule would be needed:

$$\begin{cases} \frac{b_{t+1}}{y_{t+1}} \leq \frac{b_t}{y_t} + \alpha \left(\frac{b^*}{y^*} - \frac{b_0}{y_0} \right) & \text{if} \quad \frac{b^*}{y^*} < \frac{b_0}{y_0} \\ \frac{b_{t+1}}{y_{t+1}} \leq \frac{b_t}{y_t} & \text{if} \quad \frac{b^*}{y^*} \geq \frac{b_0}{y_0} \end{cases} \tag{2}$$

However, this creates a lot of complexity in the model. In fact, preserving a rule of this form in a DSGE model rules out the possibility to solve the model linearly with a first-order perturbation approach. In turn, this implies that the model has to be solved non-linearly. To avoid dealing with such complexity, in the analysis we stick to the more straightforward linear rule letting the constraint in equation (1) always being binding.

[3]The dataset we used is available on the Eurostat website.

such as Greece, Italy, Portugal, and Ireland are required to run a considerable surplus every year to fulfill the consolidation rule. In contrast, low debt countries such as Finland, Denmark, and Sweden can simply balance their budget[4]. Figure 2 shows the debt path for selected high and low debt EU countries, respectively marked with red and blue lines. The path abstracts from GDP growth. Data until 2012 are actual realizations, while data after 2012 are forecasted assuming the dynamics described in equation (1) and shown in the grey area. In the chart, we also display the median of the 28 EU countries as a solid black line with markers and its median consolidation path. The difference between two consecutive periods in the shadow area represents the surplus that a country needs to run to fulfill the rule. The chart shows that there is a marked difference between the two clusters of countries. In particular, the high debt countries display a steep path. This feature implies that a high surplus is demanded each year. For this reason, we suppose that the full implementation of such a rule could have some relevant implications on the macroeconomic outlook of such countries. From a theoretical standpoint, our idea is that having some rigid constraints on the fiscal side may have non-negligible effects on the dynamics of other macroeconomic variables. On one side, restricting public spending may have a negative effect on output, consumption, and employment; on the other hand, it may limit the government ability to counteract exogenous shocks. To shed some lights on the topic, in the next section, we develop a general equilibrium model which includes a non-standard government sector, and we use it as a laboratory for our experiments. In particular, we augment the standard government sector included in New-Keynesian models, by adding a debt consolidation rule which mimics the one in this section. The government has to abide by a debt consolidation path, while the economy is hit by negative shocks. In this way, we are able to analyse not only the debt path but also the dynamics of all the variables in the model. Finally, we parametrize the debt consolidation rule, in a way in which we can assess the dynamics of the economy by varying the degree of tightness of the rule.

3 Model

To study whether a debt consolidation path, like the one implied by the FCR, may affect the main macroeconomic variables in the economy, we study the effects of including such provision in a fully microfounded NK model. Our interest is mainly to analyse the effects of the FCR on the economy outside the equilibrium, i.e., when the debt level has deviated from its target (which is also assumed to be the steady-state level). For this reason, our analysis mainly focuses on the impulse response functions of the system. In particular, the model we use for the analysis is a New-Keynesian model embedding infinitely life-time utility maximiser agents and monopolistically competitive firms pro-

[4]The original FCR includes the possibility to run a 1% deficit/GDP spending policy if the debt-to-GDP ratio is below 60%.

ducing differentiated goods using only labor and technology. To keep the model as smooth as possible, we do not include capital and investment. Each period, households choose between consumption and saving, and the only asset in the model is a risk-free bond issued by the government. Finally, a Central Bank is in charge of maintaining price and output stability.

Household

The household sector is made by a representative household maximising his expected lifetime utility $U(C_t, N_t)$ at period $t = 0$. We assume a utility function depending only on consumption C_t and normalised leisure $1 - N_t$. Consumers minimise expenditure given the consumption level of composite good C_t. We assume that regularity conditions on the utility function hold and that $\partial U/\partial C_t > 0$, $\partial U/\partial N_t < 0$, $\partial U/\partial C_t^2 < 0$ and $\partial U/\partial N_t^2 < 0$. Moreover, we assume a standard constant relative risk aversion (CRRA) functional form of additively separable consumption and labor.

$$\max_{C_t, N_t, B_t} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right) \tag{3}$$

Where β is the intertemporal discount factor, σ is the coefficient of the relative risk aversion, and ϕ is the inverse of the Frisch elasticity. $\mathbb{E}_0$ is the expectation operator conditional to the information set at time zero $\mathbb{E}\left[\cdot|\mathbb{I}_0\right]$ We also assume that there is a continuum (in the $[0,1]$ interval) of different goods produced with constant elasticity of substitution (CES) technology.

$$C_t = \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \tag{4}$$

Where ϵ is the parameter controlling the degree of substitutability among goods. The utility is maximised subject to the household's budget constraint, a downward-sloping demand curve, and *no-Ponzi game* condition in the government bonds market, as described in equations (5) to (7).

$$\int_0^1 P_t(i) C_t(i) di + B_t \leq (1 + R_t) B_{t-1} + W_t N_t - T_t + \Pi_t^F \tag{5}$$

$$C_t(i) = C_t \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} \tag{6}$$

$$\lim_{T \to \infty} \mathbb{E}_t \left\{ B_T \right\} \geq 0, \quad \forall t \tag{7}$$

The representative consumer allocates wealth between consumption and saving. In the equations, $P_t(i)$ denotes the prices of different goods i, $(1 + R_t)$ is the gross interest rate on risk-free bonds B_t issued by the government and purchased in the previous period. T_t is a lump-sum tax/transfer made by the government to households. W_t stands for labor price (wage) and Π_t^F captures the dividends coming from firms to households (as it is assumed that households own the firms). After some algebraic manipulations and by plugging equation (6) into the consumer's budget constraint, we can write the household mazimisation problem as a current value Lagrangian–where Λ_t is the Lagrangian multiplier. By solving the system of first-order conditions we can recover the *labour supply* (8) and the *Euler equation* (9).

$$\frac{W_t}{P_t} = N_t^{\phi} C_t^{\sigma} \tag{8}$$

$$\mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\Pi_{t+1}} \right] = \frac{1}{(1 + R_t)} \tag{9}$$

Where $\Pi_{t+1} \equiv P_{t+1}/P_t$ is the gross inflation rate.

Firms

We assume that firms operate under monopolistic competition and produce differentiated goods by using labour N_t as their only source of input. Technology A_t is equal among firms, and the production function takes the following form:

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{10}$$

Where $Y_t(i)$ stands for the production of output i and α is the output elasticity concerning labor input. In the paper, we assume that price levels adjust *à la Calvo* (Calvo, 1983) with a fraction $1 - \theta$ of re-optimizing firms and a fraction θ of non-re-optimizing firms with $\theta \in [0, 1]$. This assumption is extremely useful because it allows to compute a proxy for the potential output of the economy by setting to zero the fraction of non-re-optimizing firms. Imposing $\theta = 0$ allows removing the only source of inefficiency in the production sector by releasing the price rigidity assumption. Equation (11) displays the aggregate price index under the Calvo price assumption.

$$P_t = \left(\theta P_{t-1}^{1-\epsilon} + (1 - \theta) P_t^{*1-\epsilon} \right)^{\frac{1}{1-\epsilon}} \tag{11}$$

Where P_t^* is the optimal price chosen by the optimizing firms. As $\theta \to 0$, $P_t = P_t^*$ implying that all the firms can reset their prices as in a flexible price economy. By dividing both sides by P_{t-1}, equation (11) can also be rewritten in terms of gross inflation.

$$\Pi_t^{1-\epsilon} = \theta + (1 - \theta) \left(\frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon} \tag{12}$$

Re-optimizing firms solve a profit maximisation problem subject to a downward sloping *demand constraint*. The Lagrangian for this problem can be written as in equation (13).

$$\max_{P_t^*} \quad \mathcal{L}_3 \equiv \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left(Q_{t,t+k} \left(P_t^* \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} Y_{t+k} - \Psi_{t+k} \left(\left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} Y_{t+k} \right) \right) \right) \tag{13}$$

Where $Q_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$ is the stochastic discount factor. As we assume that households own firms, coerently we assume that the two agents have the same discount factor. Ψ_{t+k} is a cost function depending on the production level. We assume that the regularity conditions on the cost function hold. Maximising for P_t^*, we retrieve equation (14).

$$P_t^* = \mathcal{M} \frac{\sum_{k=0}^{\infty} (\theta\beta)^k \mathbb{E}_t \left[C_{t+k}^{-\sigma} Y_{t+k} P_{t+k} MC_{t+k} \right]}{\sum_{k=0}^{\infty} (\theta\beta)^k \mathbb{E}_t \left[C_{t+k}^{-\sigma} Y_{t+k} \right]} \tag{14}$$

where $\mathcal{M} = \frac{\epsilon}{\epsilon - 1}$ is the firms' mark-up. Notice that when $\theta = 0$ the optimal price setting is given by $P_t^* = \mathcal{M}\Psi'_{t|t}$. As a second-step, firms choose the optimal amount of labor to minimise their total costs subject to the resource constraint. Equations (15) and (16) display the Lagrangian function for the firms' minimisation problem and the associated first-order condition for N_t.

$$\min_{N_t(i)} \quad \mathcal{L}_4 \equiv \frac{W_t}{P_t} N_t(i) - MC_t \left(Y_t(i) - A_t N_t(i)^{1-\alpha} \right) \tag{15}$$

$$\frac{\partial \mathcal{L}_4}{\partial N_t(i)} \equiv MC_t = \frac{W_t}{P_t} \frac{1}{(1-\alpha) A_t N_t(i)^{-\alpha}} \tag{16}$$

Where MC_t is the Lagrangian multiplier, which also represents the marginal cost of increasing the production by one unit (*shadow price*). A_t is a forcing variable representing a structural shock hitting the available technology. We assume that $ln(A_t)$ is distributed as an autoregressive process of order one with normal Gaussian innovation, as in equation (17).

$$ln(A_t) = \rho_a ln(A_{t-1}) + \epsilon_{a,t}, \quad \epsilon_{a,t} \sim \mathbb{N}\left(0, \sigma_a^2\right) \tag{17}$$

Where ρ_a is the autoregressive parameter with $|\rho_a| < 1$, and $\sigma_a^2 \in \mathbb{R}^+$ is the variance of the Gaussian innovation.

Government

The government purchases a continuum of different public goods produced by firms under monopolistic competition. Like households, it maximises the consumption of public goods for any level of expenditure. Government consumption is financed by *lump-sum taxes* and abide by the *government* *budget constraint* (GBC) in real terms.

$$B_t^R = (1+R_t)\frac{B_{t-1}^R}{\Pi_t} + G_t - T_t^R \tag{18}$$

Where $B_t^R = B_t/P_t$ and $T_t^R = T_t/P_t$ are real government debt and tax revenue. As anticipated, in our model public spending is not set as a purely exogenous process, but it is composed of two different elements; the first component is deterministic and determines the debt consolidation process. The consolidation rule is a linear function of the deviation from the debt-to-GDP target. We model the deterministic part of the process according to this rule to mimic a simplified version of the EU Fiscal Compact. We call it *Fiscal Compact Rule* (FCR). The second component is a stochastic process that can be considered as an unexpected government spending shock Ω_t. We assume that $ln(\Omega_t)$ is distributed as an autoregressive process of order one with normal Gaussian innovation, in equation (20). As in the optimal fiscal policy literature, this can be thought as a war or a natural catastrophic event. We assume the same process for the tax revenue, as displayed by equation (21).

$$G_t = T_t^R + \alpha_g\left(\frac{B^{R*}}{Y^*} - \frac{B_t^R}{Y_t^F}\right) + \Omega_t \tag{19}$$

$$ln(\Omega_t) = \rho_\omega ln(\Omega_{t-1}) + \epsilon_{\omega,t}, \quad \epsilon_{\omega,t} \sim \mathbb{N}\left(0, \sigma_\omega^2\right) \tag{20}$$

$$ln(T_t^R) = \rho_\tau ln(T_{t-1}^R) + \epsilon_{\tau,t}, \quad \epsilon_{\tau,t} \sim \mathbb{N}\left(0, \sigma_\tau^2\right) \tag{21}$$

Where Y_t^F is the potential output, ρ_i is the autoregressive parameter with $|\rho_i| < 1$, and $\sigma_i^2 \in \mathbb{R}^+$ is the variance of the Gaussian innovation, with $i = \{\omega, \tau\}$. Equation (19) represents the debt consolidation rule (in real terms) that the government has to follow. The main differences between equation (1) and equation (19) is that we substitute the difference between the debt-to-GDP in two consecutive period with the primary deficit defined as $D_t \equiv P_tG_t - T_t$. Secondly, we include in the rule a stochastic process to replicate the effect of an unexpected public spending shock. As in equation (1), the term B^{R*}/Y^* is the government debt-to-GDP target exogenously set in the fiscal compact rule. We assume that the target level is equal to the steady-state level B^R/Y. This assumption will be useful in the log-linearised version of the model, where variables are described as deviation from their steady-state level. Assuming that the target level equals the steady-state level implies that the log-deviation from the steady-state is also the deviation from the target. With respect to equation (1), we also assume that the initial level of debt is computed on the basis of natural level of output instead of computing it with respect to actual output. Following Smets and Wouters (2007), in the paper we compute the natural level of output as the output prevailing in flexible prices. Including potential output into the debt consolidation process allows us to define the deficit as the *structural deficit* in the fiscal compact rule. From a dynamic perspective, including a debt consolidation policy of this form might be considered as imposing a constraint in terms of the timing of the system shock absorption. We can clarify this point using a simple example, and recalling that in a general equilibrium setting, in the absence of shocks, the variables are always at their steady-state level. Suppose that the Central Bank decides to increase the interest rate unexpectedly (*monetary policy shock*). Starting from the policy rate, all the variables in the system move away from their steady-state level. Then, including into the system equation (19) imposes a constraint on the debt dynamics and the return path to the steady-state by determining the amount of public goods that the government can purchase. This process depends on the parameter α_g which determines the velocity with which the debt has to come back to their steady-state level, and indeed the velocity with which the whole system has to come back to its steady-state. By changing the size of α_g, it is possible to change the velocity with which the shock is absorbed. It is useful to think α_g as the inverse of the number of years in which the debt reduction must be implemented. As $\alpha_g \longrightarrow 0$, the rule implies a balanced budget rule (BBR) $G_t = T_t^R$. While, as $\alpha_g \longrightarrow 1$, the target must be reached in one period by running a surplus equal to the amount of the debt in excess ED_t with respect to the target level $G_t - T_t^R = -ED_t$. By varying the parameter α_g, it is possible to explore the entire spectrum of linear debt consolidation policies which take the form of equation (19)[5].

Central Bank

We assume that a Central Bank is in charge of maintaining price and output stability by responding to output and inflation deviations from their respective targets. The feedback rule employed by the Central Bank is described in equation (22).

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi} \left(\frac{Y_t}{Y^T}\right)^{\phi_y} \Theta_t \tag{22}$$

Where Π^* and Y^T are the Central Bank targets in term of inflation and output, R is the steady-state level of the net interest rate, ϕ_π and ϕ_y are the reaction coefficients of inflation and output deviations from the targets. Finally, Θ_t is the monetary policy shock. We assume that $ln(\Theta_t)$ is distributed as an autoregressive process of order one with normal Gaussian innovation, as in equation (23).

$$ln(\Theta_t) = \rho_a ln(\Theta_{t-1}) + \epsilon_{\theta,t}, \quad \epsilon_{\theta,t} \sim \mathbb{N}\left(0, \sigma_\theta^2\right) \tag{23}$$

Where ρ_θ is the autoregressive parameter with $|\rho_\theta| < 1$, and $\sigma_\theta^2 \in \mathbb{R}^+$ is the variance of the Gaussian innovation.

[5]Even if, in the particular application, this spectrum must be reduced to the parameter space that ensures the final system of expectation difference equations to converge (Blanchard and Kahn, 1980). We are briefly introducing this problem in the next section.

3.1 Equilibrium

In the model defined by equations (8) to (14) and (16) to (23), a competitive equilibrium is defined as a vector of prices $\{P_t, R_t, W_t\}$, allocations $\{C_t, Y_t, G_t, N_t, B_t, T_t, MC_t, \Lambda_t\}$ and a process for the exogenous state variables $\{A_t, \Omega_t, T_t, \Theta_t\}$ such that:

- Households maximise the utility function subject to the budget constraint.
- Firms maximise the profit function subject to the resource constraint.
- The government abide by the budget constraint and by the fiscal compact rule.
- The Central Bank abide by the feedback rule.
- The labor and good markets clear.

The last point implies that all the output produced is also consumed, either by households or the government (equation 24), and that labor supply is equal to labor demand (equation 25).

$$Y_t = C_t + G_t \tag{24}$$

$$N_t = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di \tag{25}$$

3.2 Steady-state

The detailed steady-state relationships of the model are reported in the appendix. In what follows, we assume that variables without the time t index denote the steady-state counterpart of the dynamic variables. Thus $P_t = P$, $R_t = R$, $W_t = W$, $C_t = C$, $Y_t = Y$, $G_t = G$, $N_t = N$, $B_t = B$, $T_t = T$, $\Pi_t^F = \Pi^F$, $MC_t = MC$, $\Lambda_t = \Lambda$, $A_t = A$, $\Omega_t = \Omega$, and $\Theta_t = \Theta$.

3.3 Flexible price equilibrium

Equation (19) describes the fiscal compact rule as a function of potential output Y_t^F. Thus, we need to retrive a functional form for this variable. Starting from the firms' price maximisation, we get the flexible price mark-up $P_t^* = \mathcal{M}\Psi'_{t|t}$. Given that under flexible prices $P_t^* = P_t$ we have $MC_t = \frac{1}{\mathcal{M}}$. By plugging the flexible price mark-up into the labor demand equation $MC_t = \frac{W_t}{P_t}\frac{1}{A_t(1-\alpha)N_t^{-\alpha}}$, and using the market clearing condition $C_t = Y_t + G_t$, the labor supply $\frac{W_t}{P_t} = C_t^\sigma N_t^\phi$, and the labor market clearing condition $N_t = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}}$ we derive equation (26).

$$\left(Y_t^F - G_t^F\right)^\sigma \left(Y_t^F\right)^{\frac{\phi+\alpha}{1-\alpha}} = \mathcal{M}(1-\alpha)A_t^{\frac{1+\phi}{1-\alpha}} \tag{26}$$

Where variables (Y_t^F, G_t^F) with the superscript F denotes variables in flexible price equilibrium. To disentangle Y_t^F, we log-linearise the equation (26) around the non-stochastic steady-state. Solving for the log-linear output in flexible prices $\hat{y}_t^F = ln(Y_t^F) - ln(Y)$, we rewrite $\hat{y}_t^F$ as a function of the technology shock and government spending in log-deviation from the steady-state.

$$\hat{y}_t^F = \frac{C\,(1-\alpha)}{\sigma Y\,(1-\alpha) + C\,(\phi+\alpha)} \left(\frac{\phi+\alpha}{1-\alpha} a_t + \frac{\sigma G}{C} \hat{g}_t^F \right) \tag{27}$$

Where $a_t = ln(A_t)$, and $\hat{g}_t^F = ln(G_t^F) - ln(G)$. Finally, substituting into equation (27) the government budget constraint and fiscal compact rule in log-deviation from the steady-state in flexible-price (real terms) as described in equations (28) and (29), we derive an equation for $\hat{y}_t^F$ in terms of forcing variables, and predetermined variables. We assume that in a flexible price equilibrium, the output-gap $\tilde{y}_t$ is zero ($\tilde{y}_t = \hat{y}_t - \hat{y}_t^F$), inflation is at target level ($\hat{\pi}_t = \kappa \sum_{k=0}^{\infty} \beta^k \tilde{y}_{t+k}$) and firms can freely re-set prices ($\theta = 0$). This is in equation (30).

$$\hat{b}_t^F = \frac{(1+R)}{\Pi} \hat{b}_{t-1}^F + \frac{R}{\Pi} \theta_t + \frac{G}{B} \hat{g}_t^F - \frac{T}{B} \hat{\tau}_t \tag{28}$$

$$\hat{g}_t^F = \frac{T}{G} \hat{\tau}_t + \frac{\alpha_g B}{YG} (\hat{y}_t^F - \hat{b}_t^F) + \frac{\Omega}{G} \omega_t \tag{29}$$

$$\hat{y}_t^F = \Xi \left(\frac{\phi+\alpha}{1-\alpha} \hat{a}_t + \Xi^G \left(\frac{(\alpha_g + Y)T}{YG} \hat{\tau}_t - \frac{\alpha_g BR}{YG\Pi} \theta_t + \frac{\Omega}{G} \omega_t - \frac{\alpha_g B(1+R)}{\Pi YG} \hat{b}_{t-1}^F \right) \right) \tag{30}$$

where $\hat{y}_t = ln(Y_t) - ln(Y)$, $\hat{\pi}_t = ln(\Pi_t) - ln(\Pi)$, $\hat{b}_t^F = ln(B_t^{F,R}) - ln(B^R)$, $\hat{\tau}_t = ln(T_t^R) - ln(T^R)$, $\hat{i}_t = ln(R_t) - ln(R)$, $\hat{p}_t = ln(P_t) - ln(P)$, $\hat{p}_t^* = ln(P_t^*) - ln(P)$, $\theta_t = ln(\Theta_t)$ $\omega_t = ln(\Omega_t)$. Also, the terms $\Xi = \left(\frac{(Y+\alpha_g)C(1-\alpha)}{(\sigma Y(1-\alpha)+C(\phi+\alpha))(Y+\alpha_g)-(1-\alpha)\sigma\alpha_g B} \right)$ and $\Xi^G = \frac{\sigma GY}{C(Y+\alpha_g)}$ are convolutions of structural and steady-state coefficients.

4 Calibration

The model is calibrated on the Euro Area and comprises 28 parameters. Among these, 9 are structural parameters $\theta^S = (\sigma, \alpha_g, \beta, \phi_y, \phi_\pi, \theta, \alpha, \epsilon, \phi)$, 10 are parameters related to the forcing variables – i.e., reduced form parameters – $\theta^{RF} = (\rho_a, \rho_\zeta, \rho_\theta, \rho_\omega, \rho_\tau, \sigma_a, \sigma_\zeta, \sigma_\theta, \sigma_\omega, \sigma_\tau)$, and 9 are steady-state coefficients $\theta^{SS} = (P, \Omega, MC, R, C, G, Y, B, T)$. We select the structural parameters according to the calibration and the estimation result of Smets and Wouters (2003). The authors estimated a medium-scale DSGE model for the EA using quarterly data from the Area Wide Model and Bayesian techniques. For the calibrated parameters, we report the calibrated value, while for the estimated parameters we report the median value of the posterior distribution obtained through the Metropolis-Hastings (Metropolis et al., 1953; Hastings, 1970) sampling algorithm. The Smets and Wouters (2003)'s results are also in line with microeconomic evidence and other studies on the EA (see for example Coenen and Straub 2005). Therefore, we set the discount factor β to 0.99, to reflect an annual steady-state interest rate level of 4%, the coefficient of relative-risk aversion σ is set equal to one, collapsing $\frac{C_t^{1-\sigma}}{1-\sigma} \rightarrow ln(C_t)$ (this is also consistent with Casares 2001), the output elasticity with respect to labor $\alpha = 0.3$, and the inverse of the Frish elasticity is set to 1.18. The elasticity of substitution between different goods ϵ is set to 3.7, implying a steady-state mark-up equal to 37%. This is the average mark-up on the Euro Area for the period 1981-2004 (Christopoulou and Vermeulen, 2012). θ is set to 0.909, implying an average price duration of ten quarters. This value is sligthly higher than in other studies for the EA, as for example Galı et al. (2001), where the average price duration is set to four quarters ($\theta = 0.75$). However, our results are robust to changes in this parameter. The coefficients of the Central Bank feedback rule are set to $\phi_\pi = 1.661$ and $\phi_y = 0.143$. Setting these parameters to more standard value commonly found in the US literature ($\phi_\pi = 1.5$, $\phi_y = 0.125$) does not change our results. The parameter regulating the debt consolidation tighteness α_g is set in the baseline case to 0.05. This corresponds to an excess debt reduction to be achieved in twenty periods. As we are interested in the effects of the rule on the main macroeconomic aggregates, we solve the model for different values of α_g. In particular, we allow this parameter to range from 1 to 0.025. This corresponds to a debt reduction between 1 to 40 periods. The model is robust to variation in this parameter, and in the next section, we show the impulse responses related to these different values. For what concerns the reduced form parameters, we still refer to the median estimate in the Smets and Wouters (2003)'s article. Therefore, $\rho_a = 0.828$ and $\rho_\omega = 0.956$ with respective standard deviations equal to $\sigma_a = 0.612$ and $\sigma_\omega = 0.329$ percentage points. Also, we set $\rho_\zeta = \rho_\theta = 0$, collapsing the AR(1) processes to i.i.d. Gaussian white noises with standard deviations $\sigma_\zeta = 0.162$ and $\sigma_\zeta = 0.129$. Finally, we assume that the taxation shock has the same persistence and standard deviation of the government spending shock $\rho_\tau = \rho_\omega$ and $\sigma_\tau = \sigma_\omega$. Results are robust to changes in this assumption.

Regarding the steady-state parameters, the choice of β and ϵ pin down respectively R to 1% quarterly, and $MC = 1/\mathcal{M}$ to 0.729 (from equation B.4 in the appendix). Also, Π is equal to one as a ratio of steady-state prices. We set B/Y to 60%, corresponding to the fiscal compact rule target. This value is also consistent with other calibration of the EA as Coenen and Straub (2005). The remaining great ratios C/Y and G/Y are respectively set to 0.74 and 0.259. These values are consistent with the long-run averages of quarterly data from the Area Wide Model for private consumption and government consumption for the period 1980-2012. Also, we assume $T = G$ in steady state. From the good market clearing condition, in steady-state Y is normalized to 1 (equation B.9 in the appendix). Finally, we set $\Omega = 1$ consistent to a zero mean autoregressive process ($ln(\Omega) = 0$). Tables 3 to 5 summarise the entire set of calibrated parameters.

5 Results

In this section, we present the results from the model simulations and impulse response functions (IRFs) derived from the model.

5.1 Simulation

As outlined in the preceding sections, including a debt consolidation rule like the FCR in equation (19) imposes a constraint on the dynamics of debt towards the steady-state by determining the amount of public goods that the government can purchase. The dynamics depends on the parameter α_g which determines the velocity with which the debt returns to its steady-state level and indeed the velocity at which the whole system returns to its steady-state. In order to show the fiscal compact rule in action, we simulate $T = 100000$ data points from the model for three levels of $\alpha_g = (0.025, 0.05, 0.25)$, plus a limiting case in which $\alpha_g = 1$. Figure 3 shows the distributions of the simulated data on the left panels and the sample variance on the right panels for public debt and spending. In particular, the solid black lines show the baseline rule $\alpha_g = 0.05$, the dashed blue lines the strict rule $\alpha_g = 0.25$, the dash-dotted red lines the loose rule $\alpha_g = 0.025$, and the solid green lines with markers the limiting case in which $\alpha_g = 1$. From the left panels, we notice that the rule succeeds in shrinking the variance of the public debt and spending distributions as a direct consequence of tightening the fiscal compact rule. Moreover, from the right-hand panel, it is possible to notice that, while the effect on the public spending variance is linear, the effect on debt is exponential. To understand this concept, α_g has to be recalled as the inverse of the periods in which the debt reduction must be implemented. As $\alpha_g \to 0$, the number of periods in which the debt level has to be reabsorbed increases exponentially. As complementary information, figure 4 shows the 50 periods moving-average of a subset of the simulated data ($T_s = 2000$) for public debt and spending from the models. The figure shows how the rule shortens the oscillations of the two variables. In particular, for public debt, the period of the cycle is heavily shrunk by the rule, showing that the size of α_g is affecting the velocity of the mean-reverting behavior of the system. This feature is also evident in the curvature of the public debt impulse response functions shown in figure 5 to 9.

5.2 Impulse response analysis

Figure 5 to 9 present the impulse response functions (IRFs) of the endogenous variables due to an unexpected increase in the following forcing variables: technology a_t, cost-push ζ_t, monetary policy θ_t, government spending ω_t, and taxation τ_t. Variables are shocked one at a time by a normalised 1% surprise. In what follows, we report and describe the log-deviation from the steady-state of the following variables: output, potential output, output gap, consumption, real public debt, real government spending, inflation, nominal interest rate, real interest rate, employment (hours worked), real wage and marginal cost. For the calibrated values of the parameters, the stability conditions of the structural model are satisfied, imposing a steady-state reverting behavior to the system.

Figure 5 describes the effects of a 1% negative technology shock. First, notice that the model is robust to different calibration of α_g, and that the IRFs always reflect a similar behavior. In the four sets of impulse responses, the main differences are the impact and the persistence. However, the sign and the path are mostly unchanged across calibrations. Consistently with the economic theory, a negative technology shock depresses output and potential output. Given that potential output is more affected by the shock, the output gap is positive and inflation raises. Employment increases to maintain a constant level of production, rising wages, but at the same time consumption decreases. The Central Bank raises the policy rate to stabilize the inflation and the output gap. Given the worsening in potential output, the debt-to-GDP ratio is now higher than its target level; thus, the debt must be reduced following the FCR and by running a surplus. Analyzing the path for the three rules, we notice that as α_g increases, the curvature of the IRFs increase. This feature is evident by looking at the public debt response. When $\alpha_g = (0.25, 1)$ the hump shape dynamics of the public debt IRF is much more pronounced, while as $\alpha_g = (0.05,\ 0.025)$ the hump shape dynamics is smoother. This, in turn, implies that also the velocity with which the variable returns to its steady-state level increases.

Analysing the four sets of impulse response functions, it is worth noticing that a stricter rule makes worsens the macroeconomic framework. When α_g increases, output, employment and real wages are lower. The reason is linked to the role of the FCR. In fact, when a negative productivity shock hits the system, the debt-to-GDP ratio worsens and deviates from its target level. Then, depending on the debt consolidation rule, the government will need to run a larger/smaller surplus. When the rule is stricter, the surplus must be larger to reach the target. Running a surplus of different sizes creates a gap between the impulse response functions with different levels of α_g. When the surplus is larger, the other variables are negatively affected, and the shock is boosted out, while when the surplus can be spread in many years, this channel can be neglected. As a limiting case consider $\alpha_g = 1$, meaning that the target must be reached immediately. The solid green lines show that output, employment, and real wages are considerably lower than in the baseline case. Also, consumption is slightly lower. However, we have the presumption that this small reduction can be an artifact of the model. In fact, in the New-Keynesian macroeconomic literature, the negative correlation between consumption and public spending is a well-known puzzle, and it is mainly due to the presence of "Ricardian agents" in the model. Articles like Coenen and Straub (2005) addresses this problem by including a second type of agents beside the Ricardian households – i.e., "non-Ricardian households". We have not addressed this puzzle in the present paper. However, adding this feature would be an interesting starting point for a future extension of the model.

Figure 6 and 7 show the IRFs for a negative government spending and taxation shocks. The IRFs present dynamics similar to the technology shock, especially in the second case (figure 7). Differently, figure 6 presents a more complex scenario. In fact, the effect on potential output is now lower, and the public spending has to decrease more to abide by the fiscal compact rule. In turns, debt can return faster to the steady-state. Nevertheless, output, employment, real wages, and consumption are still lower in case of a tighter rule. Finally, figure 8 and 9 show the IRFs for a negative cost-push and monetary policy shocks. The velocity of absorption depends mainly on the calibration of the autocorrelation parameters $\rho_\zeta = \rho_\theta = 0$, which shrinks the AR process toward a Gaussian white noise. Although the response of the variables in both cases is sizable, at least for the monetary policy shock, we cannot appreciate any significant effect by varying the tightness of the fiscal compact rule. Instead, for the cost-push shock, even if the movements in the debt consolidation rule parameter cannot generate a visible variation in the impulse responses, we can appreciate a significant difference in the impact on output, employment, and real wages as for the other shocks. To quantify this point, figure 10 summarises the IRFs results. It shows the impact difference of the impulse response functions for technology, government spending, taxation and cost-push shocks for the models with $\alpha_g = 0.025$ and $\alpha_g = 1$[6]. When the difference is negative, the impact is lower for the limiting case (red bars), while when the impact is higher, the difference is positive (blue bars). The figure shows that a negative technology shock decreases real variables by more than 20 basis points under the limiting rule. The same is true for the cost-push shock. Taxation shock has milder effects, while spending presents the largest. Infact, it decreases real variables up to 70 basis points with respect to the loose case.

5.3 Sensitivity

As a final exercise, we check the sensitivity of the model to the calibrated parameters. First, for each parameter $\vartheta^{(i)} \in \theta^S \backslash \{\alpha_g\} = (\sigma, \beta, \phi_y, \phi_\pi, \theta, \alpha, \epsilon, \phi)$, we set a *plausibility range* $[\bar{\vartheta}^{(i)}, \underline{\vartheta}^{(i)}]$. Secondly, for each value in the plausibility range, we reduce the model (King and Watson, 2002), we check the saddle path stability property, and, in case a unique solution exists, we solve it (Blanchard and Kahn, 1980; King and Watson, 1998). In performing this step, we adopt a *ceteris paribus* approach – i.e., we maintain all but one the parameter fixed at the calibrated level (tables 3 to 5), and varying the remaining one. Finally, we compute the IRFs to each shock in $X_t \equiv [a_t, \zeta_t, \theta_t, \omega_t, \tau_t]$. For each couple parameter/shock, the output of the algorithm is a three-dimensional array containing at each horizon the response of each variable in $Y_t \backslash \{\hat{b}_{t-1}, \hat{b}^F_{t-1}\} \equiv \left[\hat{y}_t, \hat{\pi}_t, \hat{i}_t, \hat{b}_t, \hat{g}_t, \hat{y}^F_t, \hat{b}^F_t, \hat{g}^F_t\right]$ for each parameter in

[6] The results for the monetary policy shock are not reported as they are approximately zero.

the plausibility range. Under this approach, the IRFs distribution can be treated, on the one hand, as a measure of the model sensitivity to parameters values, on the other, as a measure of uncertainty around the true parameters. The exercise is repeated for $\alpha_g = (0.025, 0.05, 0.1, 1)$, and for each level of α_g the 5^{th} and 95^{th} percentiles of the IRFs distribution are reported. The procedure is summarised by algorithm 1. Table 6 reports the plausibility range for each parameter $\vartheta^{(i)}$.

Algorithm 1 IRFs sensitivity

for $\alpha_g = (0.025, 0.05, 0.1, 1)$ **do**
  for $\vartheta^{(i)} = [\bar{\vartheta}^{(i)}, \underline{\vartheta}^{(i)}] \in \mathbf{\Theta}^{(\mathbf{i})}$ **do**
    Reduce the model untill $rank(\mathbf{A}\left(\mathbf{\Theta}^{(\mathbf{i})}\right)) = r$, where $\mathbf{A}\left(\mathbf{\Theta}^{(\mathbf{i})}\right)$ is $(r \times r)$
    Check the model stability $|\mathbf{A}\left(\mathbf{\Theta}^{(\mathbf{i})}\right)^{-1}\mathbf{B}\left(\mathbf{\Theta}^{(\mathbf{i})}\right) - \mathbb{I}_r\boldsymbol{\lambda}| = 0$
    Solve the model $\mathbb{E}_t\mathbf{Y}_{t+1} = \mathbf{A}\left(\mathbf{\Theta}^{(\mathbf{i})}\right)^{-1}\mathbf{B}\left(\mathbf{\Theta}^{(\mathbf{i})}\right)\mathbf{Y}_t + \mathbf{A}\left(\mathbf{\Theta}^{(\mathbf{i})}\right)^{-1}\mathbf{C}\left(\mathbf{\Theta}^{(\mathbf{i})}\right)\mathbf{X}_t$
    Derive $IRFs(\mathbf{\Theta}^{(\mathbf{i})})$
  end for
  Compute $p_i^K = percentile(IRFs(\mathbf{\Theta}^{(\mathbf{i})}), K), \quad K = \{5, 95\}$
end for

Where $\mathbf{\Theta}^{(\mathbf{i})}$ stands for $\mathbf{\Theta}(\vartheta^{(\mathbf{i})})$, and $\boldsymbol{\lambda}$ are the eigenvalues of the matrix $\mathbf{A}\left(\mathbf{\Theta}^{(\mathbf{i})}\right)^{-1}\mathbf{B}\left(\mathbf{\Theta}^{(\mathbf{i})}\right)$. Figure 11 shows the IRFs for the case in which the discount factor β is the varying parameter. The impulse responses refer to a negative 1% technology shock. The example corresponds to the case in which households are more impatient than in the standard calibration of the model, and requires a higher interest rate to shift consumption across periods. The lines in the chart display the IRFs for the standard calibration of the model $\mathbf{\Theta}$, and the different levels of α_g as described in the previous paragraph. For each IRF, the areas of the respective colors highlight the intervals associated with the $(5^{th}, 95^{th})$ percentiles of the IRFs distribution computed by algorithm 1. The figure displays some mixed results. On the one hand, variables as output, debt, and spending are not affected by changes in the discount factor. On the other, variables as wages, inflation, and interest rates show some variations. This feature is particularly evident for the firsts horizons. The main finding highlighted by the chart is twofold; first, it demonstrates the robustness of the model IRFs to different calibrations of the parameters. Secondly, it confirms our main results by showing that tightening the fiscal compact rule leads to a worst macroeconomic outcome. This general pattern holds for all the shocks and structural parameters in θ^S, and their respective plausibility range. For completeness, we also display in figure 12 and 13 the shocks related to government spending and taxation. The full set of IRFs for each combination shock/parameter is reported in the online appendix.

6 Conclusion

The paper analyses the macroeconomic effects of a debt consolidation policy in the Euro Area mimicking the *Fiscal Compact Rule*. We augment a fully micro-founded New-Keynesian model with a parametric linear debt consolidation rule, and we analyse the response of the endogenous variables in the model to exogenous shocks. To fully understand the implications on the economy, we study different debt consolidation scenarios, allowing excess debt to be re-absorbed with different timing, and we show by including a debt consolidation rule that the effects of the shocks in the economy can be exacerbated . We also show that the effect of loosening or tightening the rule in response to a shock hitting nominal variables is mild, while the same variation has a more pronounced effect in case of a shock hitting real variables. Finally, we demonstrate that the macroeconomic framework worsens as a function of the rigidity of the debt consolidation rule. As a limiting case, we show that the effects on output, inflation, interest rate and employment are sizable.

For future developments, we highlight that including "non-Ricardian agents" into the model would be interesting to understand the effects of the debt consolidation rule on consumption. Naturally, also extending the model in various dimensions would benefit the analysis. In fact, as we show in the article, it is straightforward to incorporate a debt consolidation rule in a small-scale New-Keneysian model. However, including the same rule in a more extensive model bears the same difficulty level. For example, other ordinary frictions can be incorporated, as habit-persistence or wage stickiness. The same is true for an economy with a fully fledged government sector, or for an open economy model. We believe that all these extensions can be crucial and that each of these can shade some lights on the dynamics imposed by a debt consolidation rule in a New-Keynesian setting. As a final point, in the article, we thought that understanding the implications of a debt consolidation rule in the case of negative shocks was the most interesting case from an economic perspective. However, we suspect that positive and negative shocks under a debt consolidation policy rule may not have a symmetric effect. The reason is that a positive shock can help in dealing with a consolidation path creating more resources, while a negative shock absorbs additional resources and may further constrain the response of the economy to shocks. We think that understanding this point would be relevant and can provide a deeper understanding of the mechanisms relating debt constraints with exogenous shocks.

Tables and figures

Table 1: debt consolidation path implied by the fiscal compact rule.

Country	Debt/GDP	Tot. Reduction	Annual Reduction
Belgium	104.0	44.0	2.2
Bulgaria	18.0	0.0	0.0
Denmark	45.6	0.0	0.0
Germany	79.0	19.0	1.0
Estonia	9.7	0.0	0.0
Ireland	121.7	61.7	3.1
Greece	156.9	96.9	4.8
Spain	84.4	24.4	1.2
France	89.2	29.2	1.5
Italy	122.2	62.2	3.1
Cyprus	79.5	19.5	1.0
Latvia	40.9	0.0	0.0
Lithuania	39.9	0.0	0.0
Luxembourg	21.4	0.0	0.0
Hungary	78.5	18.5	0.9
Malta	67.9	7.9	0.4
Netherlands	66.5	6.5	0.3
Austria	81.7	21.7	1.1
Poland	54.4	0.0	0.0
Portugal	124.8	64.8	3.2
Romania	37.3	0.0	0.0
Slovenia	53.4	0.0	0.0
Slovakia	52.1	0.0	0.0
Finland	53.0	0.0	0.0
Sweden	36.4	0.0	0.0

Note: the table shows the debt-to-GDP ratio for the twenty-five signatory states of the "Treaty of Stability Coordination and Governance", and the "fiscal compact". Countries highlighted in red have a debt-to-GDP ratio above the target level of 60%, while those highlighted in blue have it below. The table also shows the total hypothetical reduction needed to achieve the target and the annual decrease that is necessary to reach it in twenty years.

Table 2: list of endogenous and exogenous variables.

Endogenous	Description
$\tilde{y}_t$	Output gap
$\hat{\pi}_t$	Inflation rate
$\hat{g}_t$	Public spending
$\hat{i}_t$	Policy rate
$\hat{b}_t$	Public debt
$\hat{y}_t^F$	Flex-output
$\hat{b}_t^F$	Flex-debt
$\hat{g}_t^F$	Flex-spending
Exogenous	**Description**
a_t	Technology shock
ζ_t	Cost-push shock
θ_t	Monetary policy shock
ω_t	Government spending shock
τ_t	Tax shock

Note: the table describes the endogenous and exogenous variables in the log-linearised version of the model presented in section E. The five exogenous variables follows an AR(1) process.

Table 3: structural parameters.

Parameter	Description	Value
σ	Coefficient of relative risk aversion	1.000
β	Discount factor	0.990
ϕ	Inverse of Frish elasticity	1.180
θ	Non-reoptimizing firms	0.909
ϵ	Demand elasticity	3.700
α	Output elasticity w.r.t. labour	0.300
ϕ_π	Reaction coefficient on inflation	1.661
ϕ_y	Reaction coefficient on output	0.143
α_g	Fiscal Compact debt-return rate	0.050

Note: the table summarises the structural parameters used in the baseline calibration of the model.

Table 4: steady-state parameters.

Parameter	Description	Value
C	Consumption	0.740
G	Government spending	0.259
Y	Output	1.000
B	Public debt	0.600
R	Interest rate	0.010
T	Tax revenue	0.259
MC	Marginal cost	0.729
P	Price	1.000
Ω	Gov. spending shock	1.000

Note: the table summarises the steady-state parameters used in the baseline calibration of the model.

Table 5: shock persistence and volatility.

Persistence	Description	Value
ρ_a	Productivity	0.828
ρ_ζ	Cost-push	0.000
ρ_θ	Policy rate	0.000
ρ_ω	Public spending	0.956
ρ_τ	Tax	0.956
Volatility	**Description**	**Value**
σ_a	Productivity	0.612
σ_ζ	Cost-push	0.162
σ_θ	Policy rate	0.129
σ_ω	Public spending	0.329
σ_τ	Tax	0.329

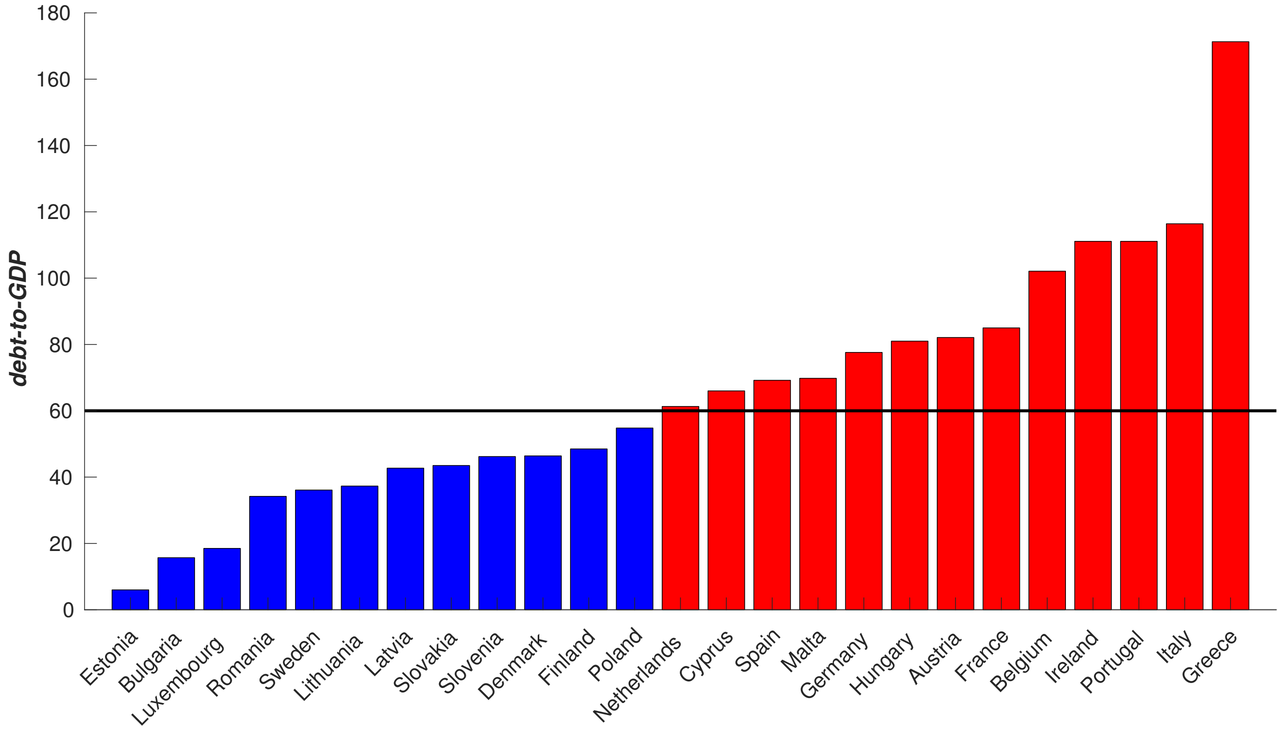
Note: the table summarises the reduced form parameters used in the baseline calibration of the AR(1) exogenous processes. Each process has two calibrated parameters, the ρ_i highlights the autocorrelation and the σ_i the standard deviation of the normal innovation in percentage points. $i = \{a, \zeta, \theta, \omega, \tau\}$.

Table 6: structural parameters.

Parameter	Standard calibration	Plausibility range
σ	1.000	[0.50, 5.00]
β	0.990	[0.50, 0.99]
ϕ	1.180	[0.50, 5.00]
θ	0.909	[0.50, 1.00]
ϵ	3.700	[2.00, 7.00]
α	0.300	[0.10, 0.90]
ϕ_π	1.661	[1.10, 2.50]
ϕ_y	0.143	[0.10, 0.50]

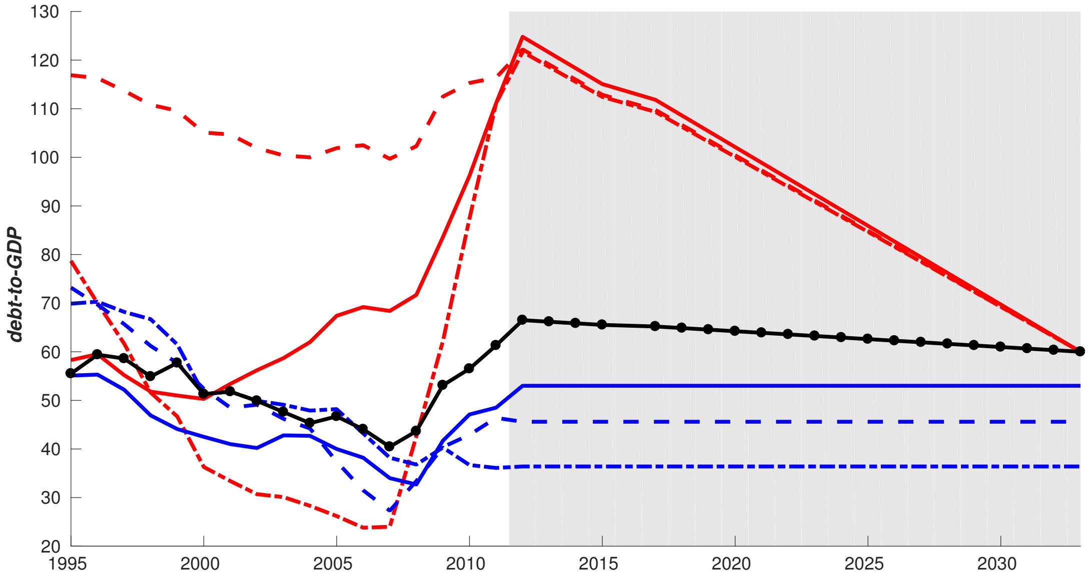
Note: the table summarises the structural parameters used in the baseline calibration of the model, and the parameter ranges used in the sensitivity analysis..

Figure 1: EU debt-to-GDP ratio.



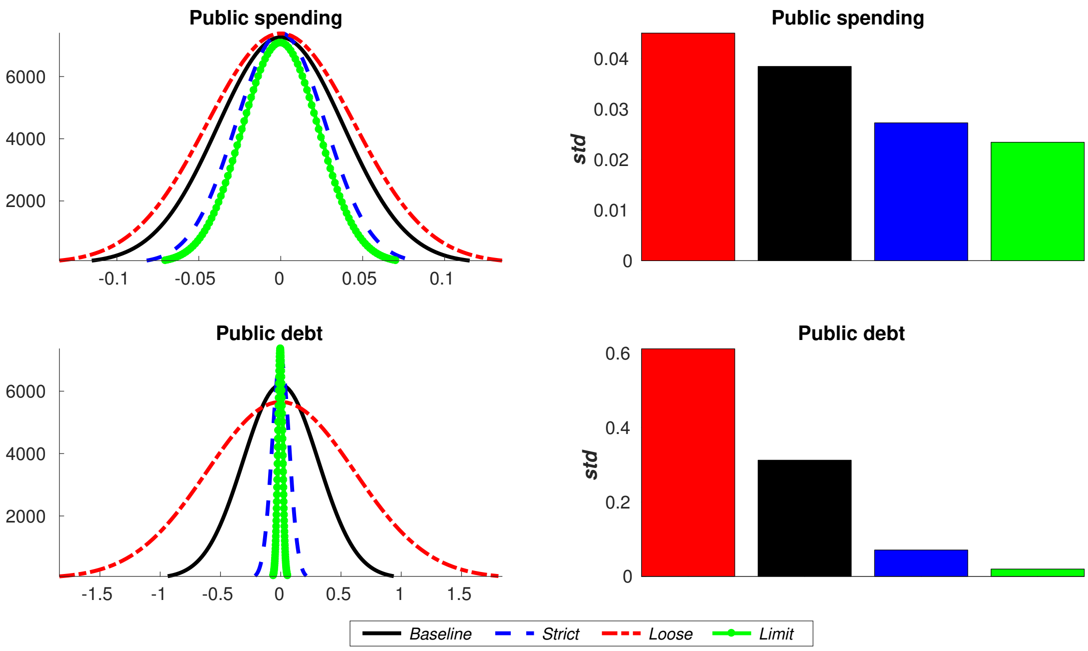
Note: the figure shows the debt-to-GDP distribution for 28 EU countries in 2012. The black solid line presents the debt-to-GDP target level according to the fiscal compact rule. Blue bars highlight countries with debt-to-GDP ratio below the target, while red bars countries with the ratio above the target.

Figure 2: predicted debt-to-GDP ratio according to the fiscal compact rule.



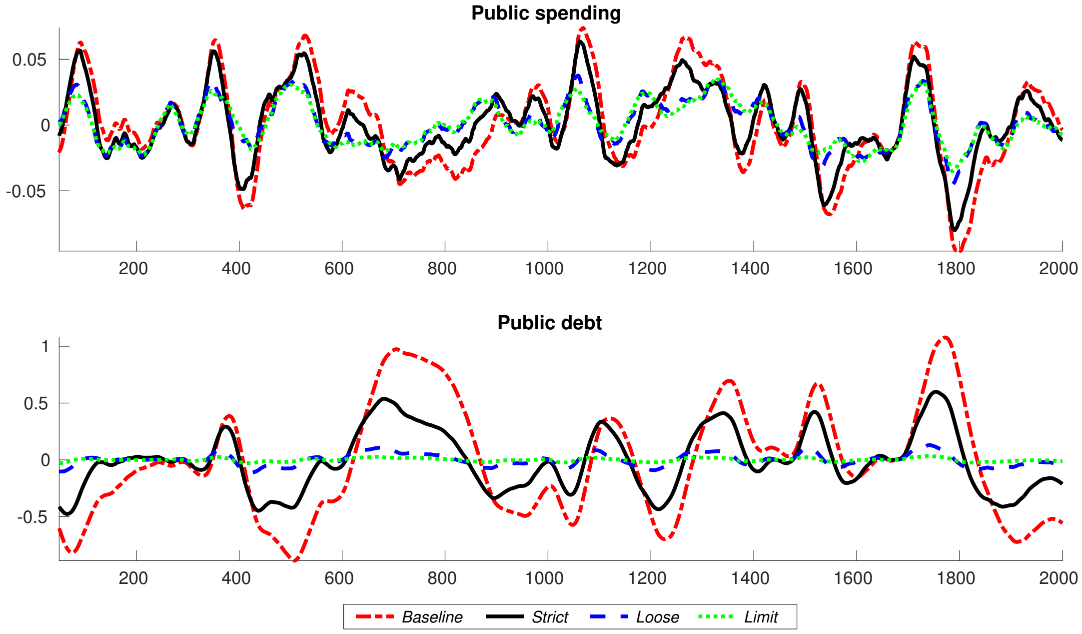
Note: the figure shows the debt-to-GDP ratio for Portugal (red-solid line), Italy (red-dashed line), Ireland (red-dash-dotted line), Finland (blue-solid line), Denmark (blue-dashed line) and Sweden (blue-dash-dotted line). Also, the black solid line with markers highlights the debt-to-GDP median of the 28 EU countries. Data before 2012 are realized, while after 2012 are computed using equation (1) and in the grey area. We set $\alpha = 0.05$ and $b^*/y^* = 60\%$.

Figure 3: Distributions and variance of public debt and spending.



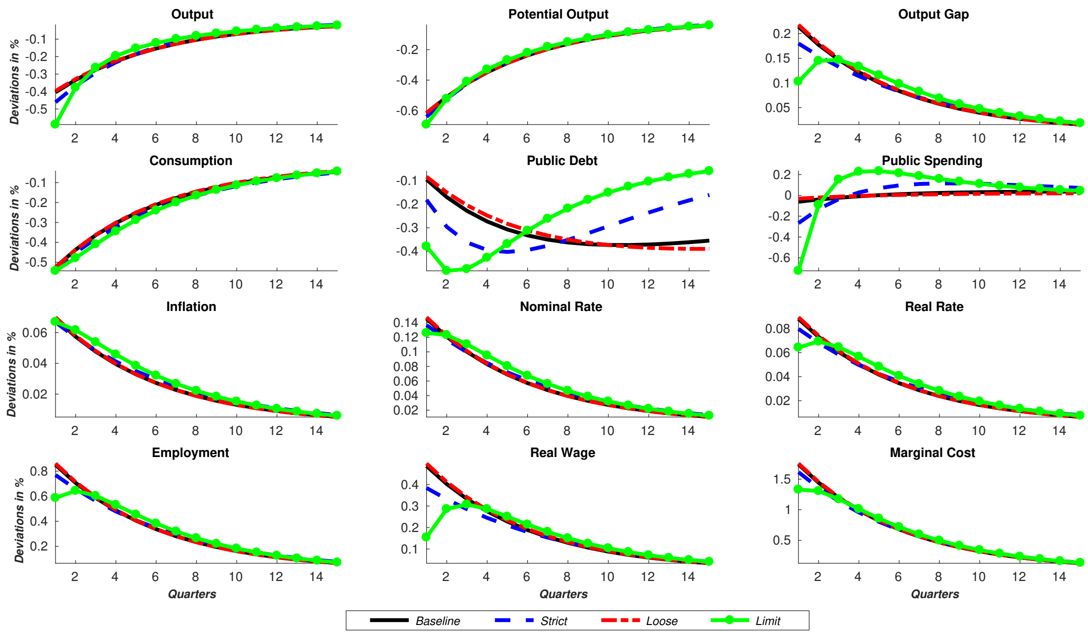
Note: on the left-hand side, the figure shows the simulated data distributions of the public debt and spending for different levels of α_g. Dash-dotted red lines show the response under the loose rule $\alpha_g = 0.025$. Solid black lines under the baseline rule $\alpha_g = 0.05$. Dashed blue lines under the strict rule $\alpha_g = 0.25$. Finally, solid green lines with markers show the limiting case in which $\alpha_g = 1$. On the right-hand side, the bar plots highlight the variance under the respective rules: loose, baseline, tight and limit.

Figure 4: Public debt and spending. Simulated series.



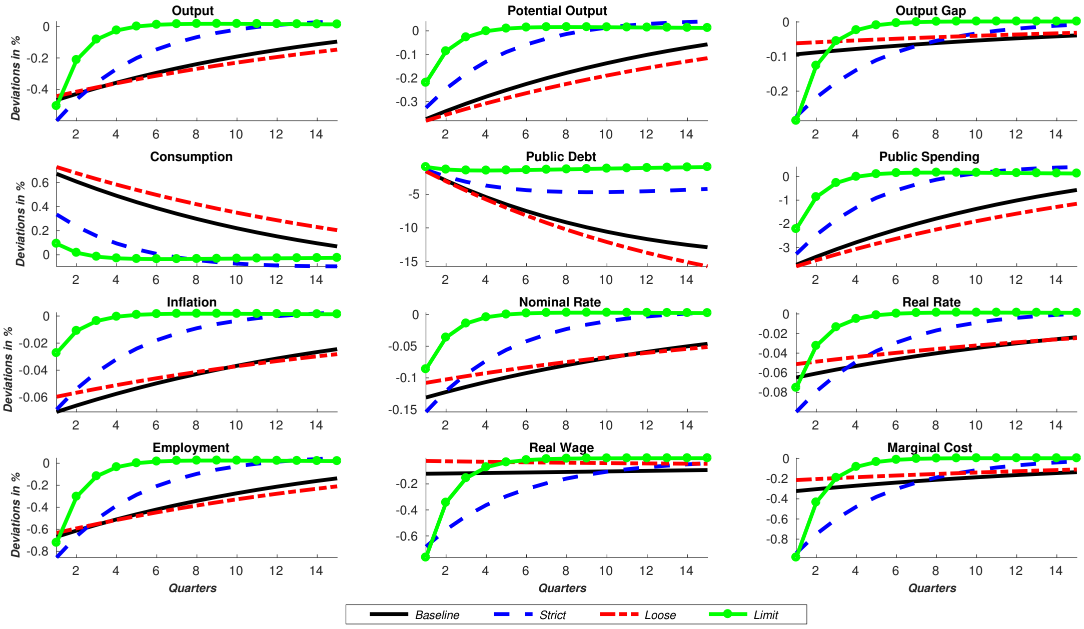
Note: the figure shows the 50 periods moving-average of 2000 simulated data points of the public debt and spending for different levels of α_g. Dash-dotted red lines show the response under the loose rule $\alpha_g = 0.025$. Solid black lines under the baseline rule $\alpha_g = 0.05$. Dashed blue lines under the strict rule $\alpha_g = 0.25$. Finally, solid green lines with markers show the limiting case in which $\alpha_g = 1$.

Figure 5: technology shock.



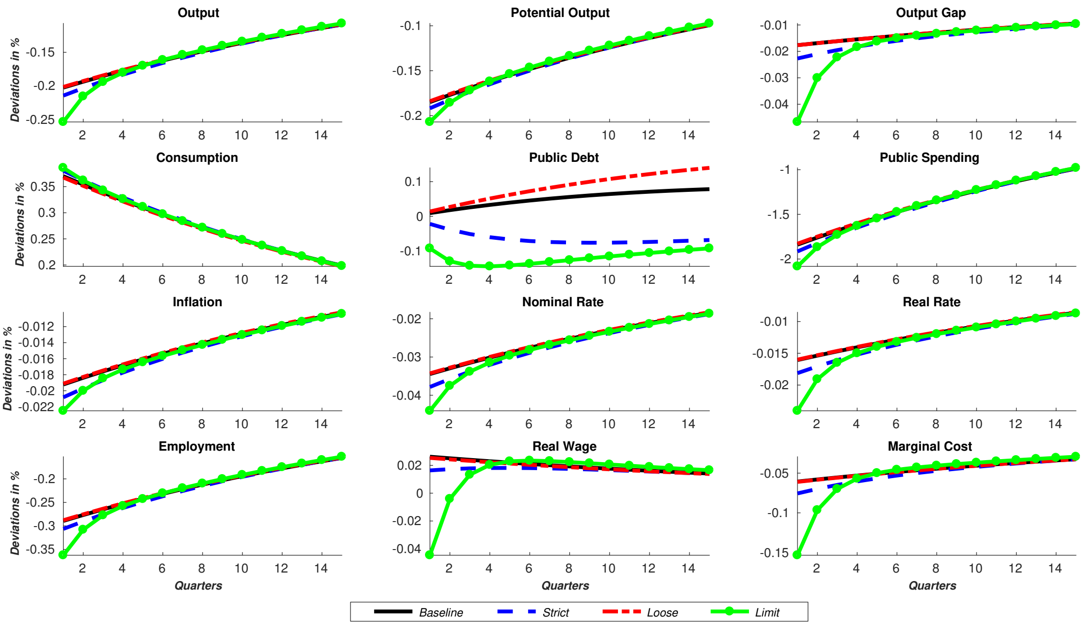
Note: the figure shows the impulse response functions to the main macroeconomic variables in the model to a negative 1% technology shock. Dash-dotted red lines show the response under the loose rule $\alpha_g = 0.025$. Solid black lines under the baseline rule $\alpha_g = 0.05$. Dashed blue lines under the strict rule $\alpha_g = 0.25$. Finally, solid green lines with markers show the limiting case in which $\alpha_g = 1$.

Figure 6: government spending shock.



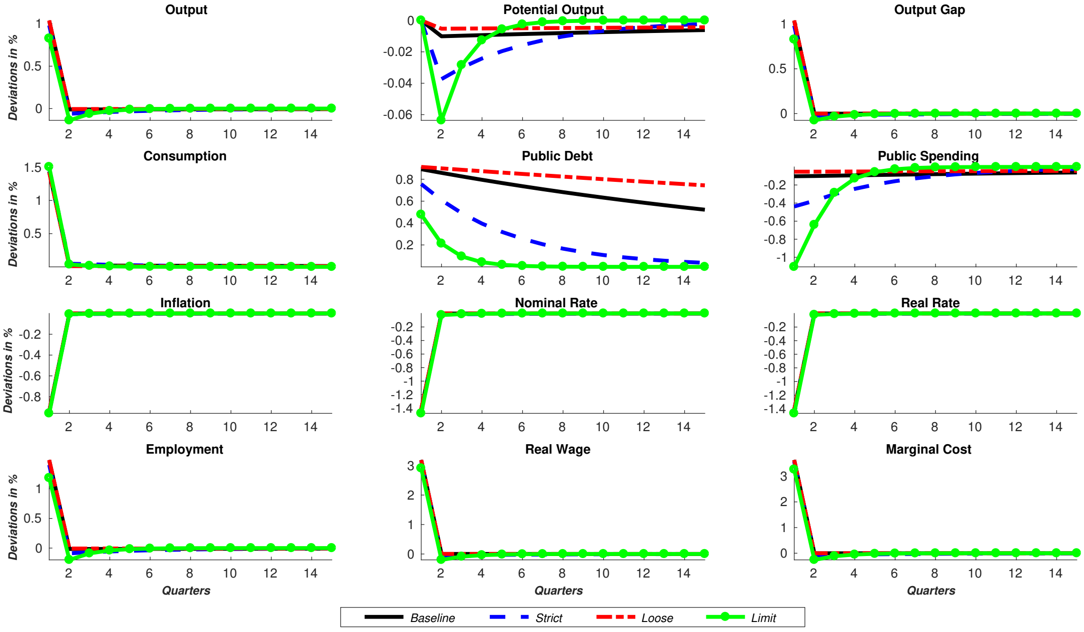
Note: the figure shows the impulse response functions to the main macroeconomic variables in the model to a negative 1% government spending shock. Dash-dotted red lines show the response under the loose rule $\alpha_g = 0.025$. *Solid black lines under the baseline rule* $\alpha_g = 0.05$. *Dashed blue lines under the strict rule* $\alpha_g = 0.25$. *Finally, solid green lines with markers show the limiting case in which* $\alpha_g = 1$.

Figure 7: tax shock.



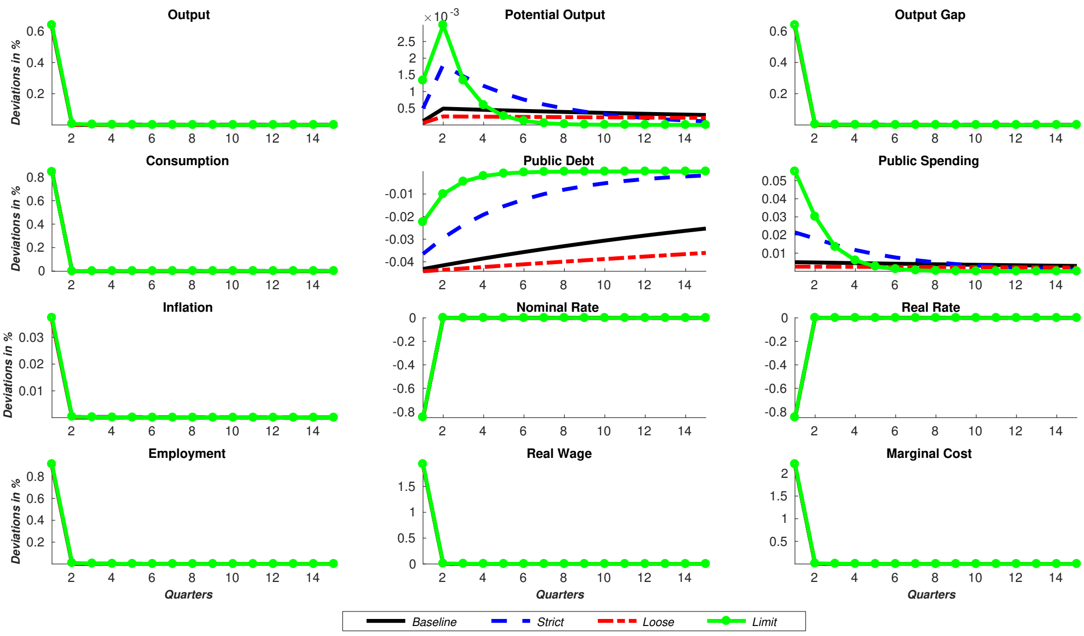
Note: the figure shows the impulse response functions to the main macroeconomic variables in the model to a negative 1% taxation shock. Dash-dotted red lines show the response under the loose rule $\alpha_g = 0.025$. *Solid black lines under the baseline rule* $\alpha_g = 0.05$. *Dashed blue lines under the strict rule* $\alpha_g = 0.25$. *Finally, solid green lines with markers show the limiting case in which* $\alpha_g = 1$.

Figure 8: cost-push shock.



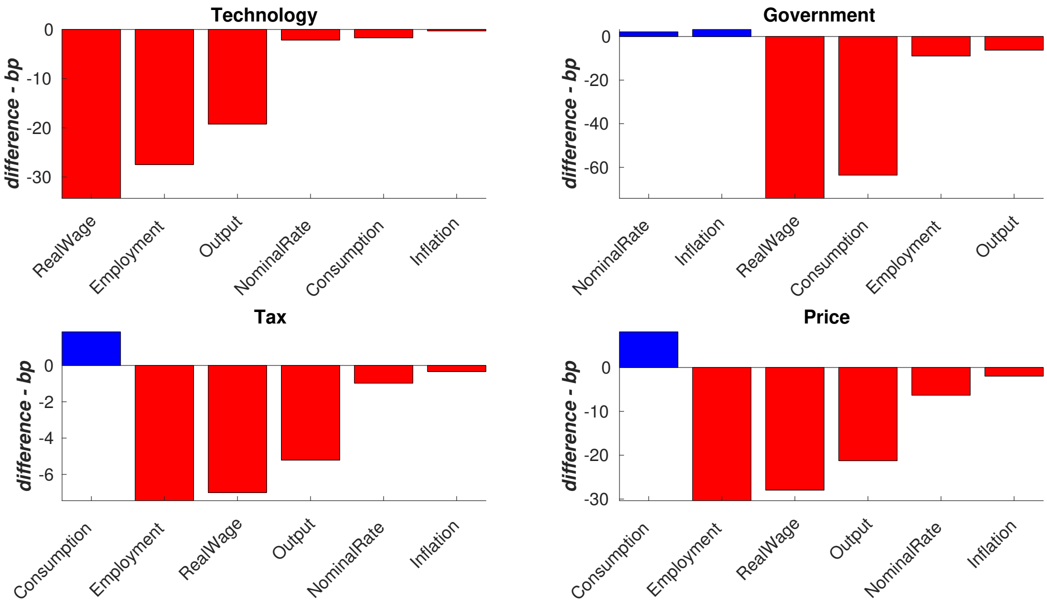
Note: the figure shows the impulse response functions to the main macroeconomic variables in the model to a negative 1% cost-push shock. Dash-dotted red lines show the response under the loose rule $\alpha_g = 0.025$*. Solid black lines under the baseline rule* $\alpha_g = 0.05$*. Dashed blue lines under the strict rule* $\alpha_g = 0.25$*. Finally, solid green lines with markers show the limiting case in which* $\alpha_g = 1$*.*

Figure 9: monetary policy shock.



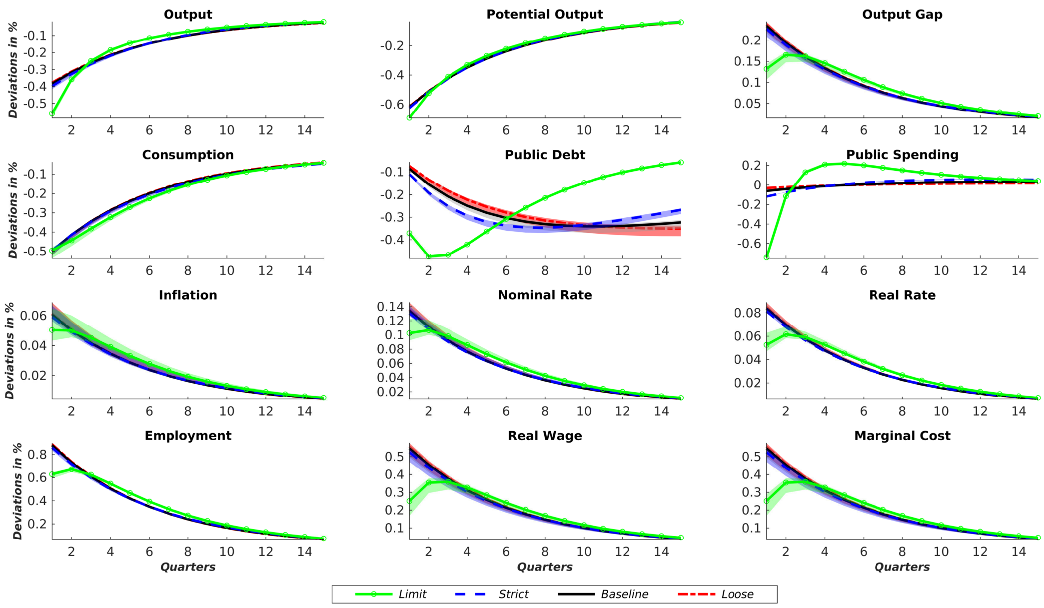
Note: the figure shows the impulse response functions to the main macroeconomic variables in the model to a negative 1% monetary policy shock. Dash-dotted red lines show the response under the loose rule $\alpha_g = 0.025$*. Solid black lines under the baseline rule* $\alpha_g = 0.05$*. Dashed blue lines under the strict rule* $\alpha_g = 0.25$*. Finally, solid green lines with markers show the limiting case in which* $\alpha_g = 1$*.*

Figure 10: IRFs impact differences.



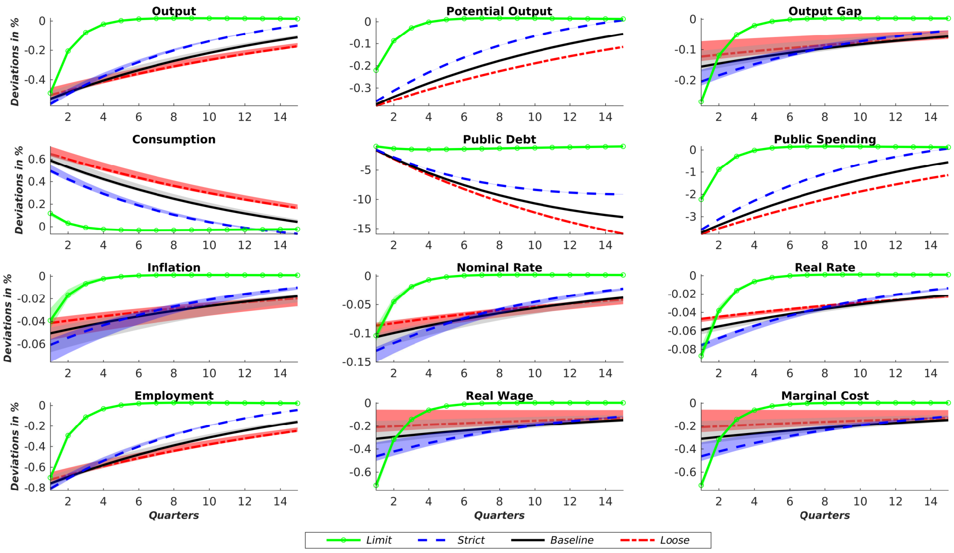
Note: the figure shows the impact differences of the IRFs for technology, government spending, taxation and cost-push shocks for models with the loose and limiting rule ($\alpha_g = 0.025$ and $\alpha_g = 1$). Red bars highlight responses for which the limiting case is lower than the loose one. Blue bars present the opposite case.

Figure 11: sensitivity. Technology shock with $\beta \in [\underline{\beta}, \bar{\beta}]$.



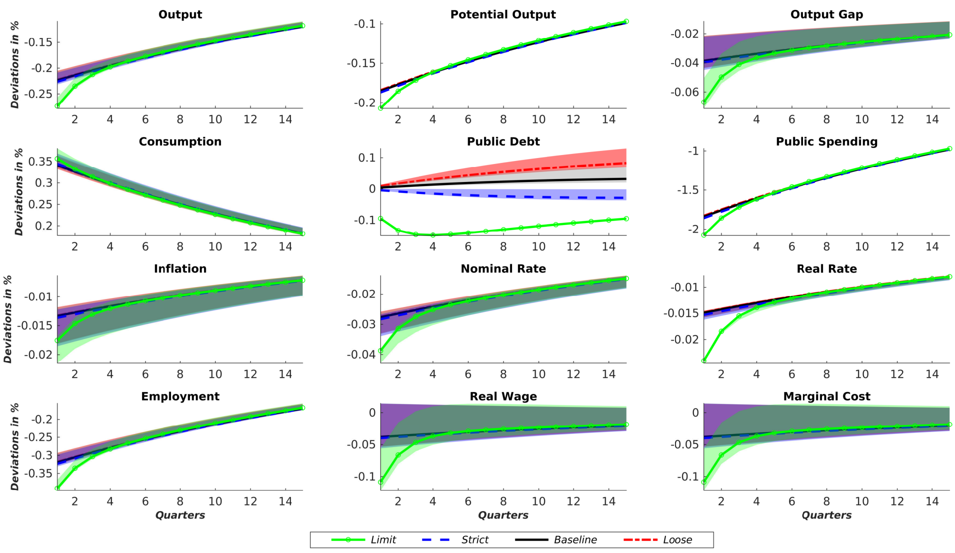
Note: the figure shows the impulse response functions to the main macroeconomic variables in the model to a negative 1% technology shock. Dash-dotted red lines show the response under the loose rule $\alpha_g = 0.025$. Solid black lines under the baseline rule $\alpha_g = 0.05$. Dashed blue lines under the strict rule $\alpha_g = 0.25$. Finally, solid green lines with markers show the limiting case in which $\alpha_g = 1$. The lines highlights the IRFs values for the baseline calibration of $\beta = 0.99$. The areas of the respective colors highlights the $(5^{th}, 95^{th})$ percentile of the IRFs distribution camputed varying $\beta \in [0.5, 0.99]$.

Figure 12: sensitivity. Government spending shock with $\beta \in [\underline{\beta}, \bar{\beta}]$.



Note: the figure shows the impulse response functions to the main macroeconomic variables in the model to a negative 1% government spending shock. Dash-dotted red lines show the response under the loose rule $\alpha_g = 0.025$*. Solid black lines under the baseline rule* $\alpha_g = 0.05$*. Dashed blue lines under the strict rule* $\alpha_g = 0.25$*. Finally, solid green lines with markers show the limiting case in which* $\alpha_g = 1$*. The lines highlights the IRFs values for the baseline calibration of* $\beta = 0.99$*. The areas of the respective colors highlights the* $(5^{th}, 95^{th})$ *percentile of the IRFs distribution camputed varying* $\beta \in [0.5, 0.99]$.

Figure 13: sensitivity. Tax shock with $\beta \in [\underline{\beta}, \bar{\beta}]$.



Note: the figure shows the impulse response functions to the main macroeconomic variables in the model to a negative 1% taxation shock. Dash-dotted red lines show the response under the loose rule $\alpha_g = 0.025$*. Solid black lines under the baseline rule* $\alpha_g = 0.05$*. Dashed blue lines under the strict rule* $\alpha_g = 0.25$*. Finally, solid green lines with markers show the limiting case in which* $\alpha_g = 1$*. The lines highlights the IRFs values for the baseline calibration of* $\beta = 0.99$*. The areas of the respective colors highlights the* $(5^{th}, 95^{th})$ *percentile of the IRFs distribution camputed varying* $\beta \in [0.5, 0.99]$.

A Appendix: system of non-linear equations

The non-linear system of equations is made-up by equations (A.1) to (A.12). These are the labor supply, Euler equation, household's budget constraint, firms optimal price setting, labor demand, price dynamics, inflation dynamics, goods market clearing, labor market clearing, government budget constraint, fiscal compact rule, Central Bank feedback rule. We also model the exogenous variables as autoregressive processes of order one. The endogenous variables are $W_t, P_t, N_t, C_t, R_t, \Pi_t, P_t^*, Y_t, MC_t, B_t, G_t, \Pi_t^F$ while A_t, Θ_t, T_t and Ω_t are exogenous forcing variables.

$$\frac{W_t}{P_t} = N_t^{\phi} C_t^{\sigma} \tag{A.1}$$

$$\mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\Pi_{t+1}} \right] = \frac{1}{(1+R_t)} \tag{A.2}$$

$$P_t C_t + B_t = (1+R_t) B_{t-1} + W_t N_t - T_t + \Pi_t^F \tag{A.3}$$

$$P_t^* = \mathcal{M} \frac{\sum_{k=0}^{\infty} (\theta\beta)^k \mathbb{E}_t \left[C_{t+k}^{-\sigma} Y_{t+k} P_{t+k} MC_{t+k} \right]}{\sum_{k=0}^{\infty} (\theta\beta)^k \mathbb{E}_t \left[C_{t+k}^{-\sigma} Y_{t+k} \right]} \tag{A.4}$$

$$MC_t = \frac{W_t}{P_t} \frac{1}{(1-\alpha) A_t N_t(i)^{-\alpha}} \tag{A.5}$$

$$P_t = \left(\theta P_{t-1}^{1-\epsilon} + (1-\theta) P_t^{*1-\epsilon} \right)^{\frac{1}{1-\epsilon}} \tag{A.6}$$

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta) \left(\frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon} \tag{A.7}$$

$$Y_t = C_t + G_t \tag{A.8}$$

$$N_t = \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\epsilon}{1-\alpha}} di \tag{A.9}$$

$$B_t = (1+R_t) B_{t-1} + P_t G_t - T_t \tag{A.10}$$

$$G_t = \frac{T_t}{P_t} + \alpha_g \left(\frac{B^*}{P^* Y^*} - \frac{B_t}{P_t Y_t^F} \right) + \Omega_t \tag{A.11}$$

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi} \left(\frac{Y_t}{Y^T}\right)^{\phi_y} \Theta_t \tag{A.12}$$

B Appendix: steady-state relationships

In steady-state we obtain the relationships described by equations (B.1) to (B.12) by dropping the time subscript.

$$\frac{W}{P} = N^{\phi} C^{\sigma} \tag{B.1}$$

$$R = \frac{1-\beta}{\beta} \tag{B.2}$$

$$PC = RB + WN - T + \Pi^F \tag{B.3}$$

$$MC = \frac{1}{\mathcal{M}} \tag{B.4}$$

$$MC = \frac{W}{P} \frac{1}{(1-\alpha)N^{-\alpha}} \tag{B.5}$$

$$P^* = P \tag{B.6}$$

$$\Pi = 1 \tag{B.7}$$

$$Y = C + G \tag{B.8}$$

$$N = \left(\frac{Y}{A}\right)^{\frac{1}{1-\alpha}} \tag{B.9}$$

$$B = \frac{1}{R}(PG - T) \tag{B.10}$$

$$G = \frac{T}{P} + \Omega \tag{B.11}$$

$$\Theta_t = 1 \tag{B.12}$$

C Appendix: real government budget constraint

It is a well-known issue in New-Keynesian models that price level may exhibit a non-stationary behavior, leading to unitary eigenvalues and unit root processes. To address this issue, and remove from the final system of equations any dependency from the price level, we rewrite the government budget constraint and the fiscal compact rule in real terms. By dividing/multiplying by P_t and P_{t-1} equation (A.10), and after some algebraic manipulations, we retrieve equation (C.2).

$$\frac{B_t}{P_t} = (1 + R_t)\frac{B_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t} + G_t - \frac{T_t}{P_t} \tag{C.1}$$

$$B_t^R = (1 + R_t)\frac{B_{t-1}^R}{\Pi_{t-1}} + G_t - T_t^R \tag{C.2}$$

The fiscal compact rule in real terms is ready available by collecting terms in equation (C.3).

$$G_t = T_t^R + \alpha_g \left(\frac{B^{R*}}{Y^*} - \frac{B_t^R}{Y_t^F}\right) + \Omega_t \tag{C.3}$$

D Appendix: log-linearisation

Model log-linearisation mainly follows the three Uhlig (1995) building blocks, reported in equations (D.1) to (D.3).

$$e^{\hat{x}_t + a\hat{y}_t} \approx 1 + \hat{x}_t + a\hat{y}_t \tag{D.1}$$

$$\hat{x}_t\hat{y}_t \approx 0 \tag{D.2}$$

$$\mathbb{E}_t\left[ae^{\hat{x}_{t+1}}\right] \approx \mathbb{E}_t\left[a\hat{x}_{t+1}\right] \tag{D.3}$$

Where $\hat{x}_t$ and $\hat{y}_t$ are real variables with values close to zero and defined as $\hat{x}_t = ln(X_t) - ln(X)$ and $\hat{y}_t = ln(Y_t) - ln(Y)$. a represents a constant, however, the second and third building blocks are defined up to a constant. As suggested by Uhlig we replace each variable by $Xe^{\hat{x}_t}$, than applying the three building blocks. After some manipulations, all the constants drop out to each equations. The fully-log-linearised model is reported in equations (D.4) to (D.16).

$$\hat{w}_t - \hat{p}_t = \phi\hat{n}_t + \sigma\hat{c}_t \tag{D.4}$$

$$\hat{c}_t = \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\sigma}\left(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}\right) \tag{D.5}$$

$$\hat{\pi}_t = (1-\theta)(\hat{p}_t^* - \hat{p}_{t-1}) \tag{D.6}$$

$$\sum_{k=0}^{\infty} \theta^k \beta^k \left(\hat{p}_t^* - \hat{p}_{t-1}\right) = \sum_{k=0}^{\infty} \theta^k \beta^k \mathbb{E}_t \left[\left(\hat{mc}_{t+k|t} + (\hat{p}_{t+k} - \hat{p}_{t-1})\right)\right] \tag{D.7}$$

$$\hat{c}_t = \frac{Y}{C}\hat{y}_t - \frac{G}{C}\hat{g}_t \tag{D.8}$$

$$\hat{n}_t = \frac{1}{(1-\alpha)}(\hat{y}_t - \hat{a}_t)^{7} \tag{D.11}$$

$$\hat{mc}_t = \hat{w}_t - \hat{p}_t - \hat{a}_t + \alpha \hat{n}_t \tag{D.12}$$

$$\hat{p}_t = (1-\theta)\hat{p}_t^* + \theta \hat{p}_{t-1} \tag{D.13}$$

$$\hat{b}_t = \frac{(1+R)}{\Pi}\left(\hat{b}_{t-1} - \hat{\pi}_t\right) + \frac{R}{\Pi}\hat{i}_t + \frac{G}{B}\hat{g}_t - \frac{T}{B}\hat{\tau}_t \tag{D.14}$$

$$\hat{g}_t = \frac{T}{G}\hat{\tau}_t + \frac{\alpha_g B}{YG}(\hat{y}_t^F - \hat{b}_t) + \frac{\Omega}{G}\omega_t \tag{D.15}$$

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_y \tilde{y}_t + \theta_t \tag{D.16}$$

D.1 Appendix: minimum set of equations

To simplify the model to a minimum set of equations, we mainly follow Gali (2008). Recalling the convergence result for geometric series $\frac{1}{\sum_{k=0}^{\infty} \theta^k \beta^k} = (1 + \beta\theta)$, equation (D.7) can be rewritten as

[7] Taking the log-deviation of the market clearing condition we get

$$N(1+\hat{n}) = \left(\frac{Y}{A}\right)^{\frac{1}{1-\alpha}} \left(1 + \left(\frac{1}{1-\alpha}\right)(\hat{y}_t - \hat{a}_t)\right) + RES \tag{D.10}$$

where RES is a very small quantity in a neighborhood of the zero inflation steady-state and can be neglected in a first order Taylor expansion. See Gali (2008) chapter 3, Appendix 3.3.

$$\hat{y}_t = (1-\alpha)\,\hat{n}_t + \hat{a}_t \tag{D.11}$$

(D.17).

$$\hat{p}_t^* - \hat{p}_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}\theta^k\beta^k\mathbb{E}_t\left[\left(\hat{mc}_{t+k|t} + (\hat{p}_{t+k} - \hat{p}_{t-1})\right)\right] \tag{D.17}$$

As $\alpha \neq 0$, we rule out the constant return to scale hypothesis, meaning $\hat{mc}_{t+k|t} \neq \hat{mc}_{t+k}$. We than need to find an equation for $\hat{mc}_{t+k|t}$. Starting from the marginal cost equation and plugging $\hat{n}_t = \frac{1}{1-\alpha}(\hat{a}_t - \alpha\hat{y}_t)$ we derive an equation for $\hat{mc}_{t+k}$, and accordingly for $\hat{mc}_{t+k|t}$ – equations (D.18) and (D.19).

$$\hat{mc}_{t+k} = \hat{w}_{t+k} - \hat{p}_{t+k} - \frac{1}{1-\alpha}(\hat{a}_{t+k} - \alpha\hat{y}_{t+k}) \tag{D.18}$$

$$\hat{mc}_{t+k|t} = \hat{w}_{t+k} - \hat{p}_{t+k} - \frac{1}{1-\alpha}(\hat{a}_{t+k} - \alpha\hat{y}_{t+k|t}) \tag{D.19}$$

Thus, equating the two expressions, and by plugging the demand schedule, we derive an expression for $\hat{mc}_{t+k|t}$[8].

$$\hat{mc}_{t+k|t} - \hat{mc}_{t+k} = \frac{\alpha}{1-\alpha}(\hat{y}_{t+k|t} + \hat{y}_{t+k}) \tag{D.20}$$

$$\hat{mc}_{t+k|t} = \hat{mc}_{t+k} + \frac{\alpha\epsilon}{1-\alpha}\left(\hat{p}_t^* - \hat{p}_{t+k}\right) \tag{D.21}$$

Finally, by plugging (D.21) into (D.17), and after some algebraic manipulations, using $\hat{\pi}_t = (1-\theta)(\hat{p}_t^* - \hat{p}_{t-1})$, we can retrieve an equation for the New-Keynesian Phillips curve.

$$\hat{p}_t^* - \hat{p}_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}\theta^k\beta^k\mathbb{E}_t\left(mc_{t+k} - \frac{\alpha\epsilon}{1-\alpha}\left(\hat{p}_t^* - \hat{p}_{t+k}\right) + \left(\hat{p}_{t+k} - \hat{p}_{t-1}\right)\right) \tag{D.22}$$

$$\hat{p}_t^* - \hat{p}_{t-1} = (1-\beta\theta)\,\Theta\sum_{k=0}^{\infty}\theta^k\beta^k\mathbb{E}_t mc_{t+k} + \sum_{k=0}^{\infty}\theta^k\beta^k\mathbb{E}_t\left(\hat{p}_{t+k} - \hat{p}_{t-1}\right) \tag{D.23}$$

$$\hat{p}_t^* - \hat{p}_{t-1} = (1-\beta\theta)\,\Theta\frac{1}{(1-\theta\beta F)}mc_t + \frac{1}{(1-\theta\beta F)}\left(\hat{p}_t - \hat{p}_{t-1}\right) \tag{D.24}$$

$$\hat{p}_t^* - \hat{p}_{t-1} = \beta\theta\mathbb{E}_t(\hat{p}_{t+1}^* - \hat{p}_t) + (1-\beta\theta)\,\Theta\hat{mc}_t + \hat{\pi}_t \tag{D.25}$$

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \lambda\hat{mc}_t \tag{D.26}$$

Where $\Theta = \frac{1-\alpha}{1-\alpha-\alpha\epsilon}$, $\lambda = \frac{(1-\theta)(1-\beta\theta)\Theta}{\theta}$. and F is the forward operator. Finally, after some manipulations, the marginal cost equation is given by plugging into (D.13) the labour supply $\hat{w}_t - \hat{p}_t =$

[8] By taking log of the *demand constraint* $Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon}(Y_{t+k})$ we have $\hat{y}_{t+k|t} = \hat{y}_{t+k} + \epsilon\left(\hat{p}_t^* - \hat{p}_{t+k}\right)$. Then $\hat{mc}_{t+k} = \hat{mc}_{t+k|t} + \frac{\alpha\epsilon}{1-\alpha}\left(\hat{p}_t^* - \hat{p}_{t+k}\right)$.

$\frac{\sigma Y}{C}\hat{y}_t - \frac{\sigma G}{C}\hat{g}_t + \phi\hat{n}_t$ and the log of the market clearing conditions $\hat{n}_t = \frac{1}{1-\alpha}\left(\hat{y}_t - \hat{a}_t\right)$.

$$\hat{mc}_t = \left(\frac{\sigma Y(1-\alpha) + C(\phi+\alpha)}{C(1-\alpha)}\right)\hat{y}_t - \left(\frac{\phi+1}{1-\alpha}\right)\hat{a}_t - \frac{\sigma G}{C}\hat{g}_t \tag{D.27}$$

Re-arranging in a convenient way, and by plugging the log-linear equation of flexible equilibrium output we derive equation (D.30)[9].

$$\begin{aligned}\hat{mc}_t = &\left(\frac{\sigma Y(1-\alpha) + C(\phi+\alpha)}{C(1-\alpha)}\right)\\ &\left(\hat{y}_t - \left(\frac{C(1-\alpha)}{\sigma Y(1-\alpha) + C(\phi+\alpha)}\right)\left(\left(\frac{\phi+1}{1-\alpha}\right)\hat{a}_t - \frac{\sigma G}{C}\hat{g}_t\right)\right)\end{aligned} \tag{D.28}$$

$$\hat{mc}_t = \left(\frac{\sigma Y(1-\alpha) + C(\phi+\alpha)}{C(1-\alpha)}\right)\left(\hat{y}_t - \hat{y}_t^F\right) \tag{D.29}$$

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa \tilde{y}_t + \zeta_{\pi,t} \tag{D.30}$$

where $\kappa = \lambda \frac{\sigma Y(1-\alpha)+C(\phi+\alpha)}{C(1-\alpha)}$, while $\zeta_t = \rho_\zeta \zeta_{t-1} + \epsilon_{\zeta,t}$ is the cost-push shock.

Secondly, to find a functional form for the output-gap on the form of the New-Keynesian IS curve, we exploit the Euler equation. Recalling the log form of the Euler equation we have $\hat{c}_t = \mathbb{E}_t\hat{c}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1})$. Plugging the market-clearing condition for the goods market we have the following relationships:

$$\hat{y}_t = \mathbb{E}_t\hat{y}_{t+1} - \frac{C}{Y\sigma}(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1}) - \frac{G}{Y}(\mathbb{E}_t\hat{g}_{t+1} - \hat{g}_t) \tag{D.31}$$

To write it as function of the output-gap, we sum and subtract the flexible price output y_t^F.

$$\tilde{y}_t = \mathbb{E}_t\tilde{y}_{t+1} - \frac{C}{Y\sigma}(\hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1}) - \frac{G}{Y}(\mathbb{E}_t\hat{g}_{t+1} - \hat{g}_t) + \epsilon_{y,t} \tag{D.32}$$

Where $\epsilon_{y,t} = \hat{y}_{t+1}^F - \hat{y}_t^F$.

E Appendix: the log-linearised model

The final model described by equations (8) to (14) and (16) to (24) is fully non-linear and cannot be solved analytically. Numerical computation methods, as the projection technique is required to approximate the non-linear policy function, and solving the model. However, as we derived the flexible output equation in a log-linear fashion, the most straightforward route to solve the model is by fully log-linearising it around the non-stochastic steady-state, and taking a first-order perturbation approach. In this section, we summarise the log-linearised version of the model, described by equations

[9] $y_t^F = \left(\frac{C(1-\alpha)}{\sigma Y(1-\alpha)+C(\phi+\alpha)}\right)\left(\left(\frac{\phi+\alpha}{1-\alpha}\right)\hat{a}_t + \frac{\sigma G}{C}\hat{g}_t\right)$

(E.33) to (E.41). The full log-linearisation procedure is reported in the appendix.

$$\tilde{y}_t = \mathbb{E}_t\tilde{y}_{t+1} - \frac{CY}{\sigma}(\hat{i}_t - \mathbb{E}_t\pi_{t+1}) - \frac{G}{Y}(\mathbb{E}_t\hat{g}_{t+1} - \hat{g}_t) + \hat{\epsilon}_{y,t} \tag{E.33}$$

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \kappa\tilde{y}_t + \zeta_t \tag{E.34}$$

$$\hat{i}_t = \phi_\pi\hat{\pi}_t + \phi_y\tilde{y}_t + \theta_t \tag{E.35}$$

$$\hat{b}_t = \frac{(1+R)}{\Pi}\left(\hat{b}_{t-1} - \hat{\pi}_t\right) + \frac{R}{\Pi}\hat{i}_t + \frac{G}{B}\hat{g}_t - \frac{T}{B}\hat{\tau}_t \tag{E.36}$$

$$\hat{g}_t = \frac{T}{G}\hat{\tau}_t + \frac{\alpha_g B}{YG}(\hat{y}_t^F - \hat{b}_t) + \frac{\Omega}{G}\omega_t \tag{E.37}$$

$$\hat{y}_t^F = \Xi\left(\frac{\phi+\alpha}{1-\alpha}\hat{a}_t + \Xi^G\left(\frac{(\alpha_g+Y)T}{YG}\hat{\tau}_t - \frac{\alpha_g BR}{YG\Pi}\theta_t + \frac{\Omega}{G}\omega_t - \frac{\alpha_g B(1+R)}{\Pi YG}\hat{b}_{t-1}^F\right)\right) \tag{E.38}$$

$$\hat{b}_t^F = \frac{(1+R)}{\Pi}\hat{b}_{t-1}^F + \frac{R}{\Pi}\theta_t + \frac{G}{B}\hat{g}_t^F - \frac{T}{B}\hat{\tau}_t \tag{E.39}$$

$$\hat{g}_t^F = \frac{T}{G}\hat{\tau}_t + \frac{\alpha_g B}{YG}(\hat{y}_t^F - \hat{b}_t^F) + \frac{\Omega}{G}\omega_t \tag{E.40}$$

$$\hat{\epsilon}_{y,t} = \mathbb{E}_t\hat{y}_{t+1}^F - \hat{y}_t^F \tag{E.41}$$

Where $\kappa = \lambda\frac{\sigma Y(1-\alpha)+C(\phi+\alpha)}{C(1-\alpha)}$, $\lambda = \Theta^\lambda\frac{(1-\theta)(1-\beta\theta)}{\theta}$, and $\Theta^\lambda = \frac{1-\alpha}{1-\alpha-\alpha\epsilon}$. Also, ζ_t is a forcing variable directly hitting the inflation level as in Smets and Wouters (2003). We assume that ζ_t is distributed as an autoregressive process of order one with normal Gaussian innovation $\zeta_t = \rho_\zeta\zeta_{t-1} + \epsilon_{\zeta,t}$, where $\epsilon_{\zeta,t} \sim \mathbb{N}(0, \sigma_\zeta^2)$, ρ_ζ is the autoregressive parameter with $|\rho_\zeta| < 1$, and $\sigma_\zeta^2 \in \mathbb{R}^+$ is the variance of the Gaussian innovation.

F Appendix: solution method

The log-linearised model can be solved via different methodologies, as it results in a system of linear stochastic difference equations under rational expectations (Blanchard and Kahn, 1980; King and Watson, 1998, 2002; Sims, 2002). Although these methodologies return approximately the same solution, they are tailored for different issues that can arise in the model specification. We employ the technique developed by King and Watson (1998, 2002), and in this section, we briefly describe their methodology. With some minor differences, all the methodologies start rewriting the system of difference equations in companion form, as shown in equation (F.42).

$$\mathbf{A}(\mathbf{\Theta})\,\mathbb{E}_t\mathbf{Y}_{t+1} = \mathbf{B}(\mathbf{\Theta})\,\mathbf{Y}_t + \mathbf{C}(\mathbf{\Theta})\,\mathbf{X}_t \tag{F.42}$$

Where $\mathbf{A}(\mathbf{\Theta})$ is the matrix of coefficients related to the forward-looking endogenous variables, $\mathbf{B}(\mathbf{\Theta})$ is the matrix of coefficients related to the predetermined variables and backward-looking variables; finally, $\mathbf{C}(\mathbf{\Theta})$ is the matrix of coefficients of the exogenous variables. These matrices depend on structural parameters, steady-state values and reduced form parameters collected in the vector $\mathbf{\Theta}$. Also, $\mathbf{Y}_{t+1}$ is a vector of forward-looking endogenous variables, while $\mathbf{Y}_t$ is a vector of backward-looking and predetermined variables. $\mathbf{X}_t$ is a vector of exogenous variables distributed as an AR(1) and dependent on a set of i.i.d. exogenous shock. In our case, the choice of the King and Watson (1998) solution methods is because the conditions necessary to solve the system are not fulfilled (Blanchard and Kahn, 1980). The conditions require the invertibility of the leading matrix $\mathbf{A}(\mathbf{\Theta})$ to solve the companion form for $\mathbb{E}_t\mathbf{Y}_{t+1}$, as in equation (F.43).

$$\mathbb{E}_t\mathbf{Y}_{t+1} = \mathbf{A}(\mathbf{\Theta})^{-1}\,\mathbf{B}(\mathbf{\Theta})\,\mathbf{Y}_t + \mathbf{A}(\mathbf{\Theta})^{-1}\,\mathbf{C}(\mathbf{\Theta})\,\mathbf{X}_t \tag{F.43}$$

To deal with this problem, we apply the generalized version of the Blanchard and Kahn (1980) methodology due to King and Watson (1998, 2002). In the baseline case, we have eight endogenous variables, two predetermined variables and five shocks. These variables are then collected in the following vectors:

- $\mathbb{E}_t Y_{t+1} \equiv \mathbb{E}_t\left[\hat{y}_{t+1}, \hat{\pi}_{t+1}, \hat{i}_{t+1}, \hat{b}_{t+1}, \hat{g}_{t+1}, \hat{y}^F_{t+1}, \hat{b}^F_{t+1}, \hat{g}^F_{t+1}, \hat{b}_t, \hat{b}^F_t\right]$
- $Y_t \equiv \left[\hat{y}_t, \hat{\pi}_t, \hat{i}_t, \hat{b}_t, \hat{g}_t, \hat{y}^F_t, \hat{b}^F_t, \hat{g}^F_t, \hat{b}_{t-1}, \hat{b}^F_{t-1}\right]$
- $X_t \equiv [a_t, \zeta_t, \theta_t, \omega_t, \tau_t]$

In what follows, the leading matrix $\mathbf{A}(\mathbf{\Theta})$ is decomposed to achieve invertibility. Thus, $\mathbf{A}(\mathbf{\Theta})$ and $\mathbf{B}(\mathbf{\Theta})$ can be rewritten as in equation (F.44) – where the dependence from the vector $\mathbf{\Theta}$ is not reported for easyness of reading.

$$\mathbf{A} = \mathbf{Q}'\Omega_e\mathbf{Z}', \quad \mathbf{B} = \mathbf{Q}'\Lambda_e\mathbf{Z}' \tag{F.44}$$

Where $\mathbf{Q}$ and $\mathbf{Z}$ are "unitary" matrices ($\mathbf{Q}\mathbf{Q}' = \mathbf{Q}'\mathbf{Q} = \mathbb{I}$) and Ω_e and Λ_e are upper-triangular matrices containing the generalized eigenvalues. Than the system can be "de-coupled" into a stable and an unstable part.

$$\mathbb{E}_t\mathbf{Z}_{t+1} = \Lambda_e^{-1}\Omega_e\mathbf{Z}_t + \mathbf{R}\mathbf{X}_t \tag{F.45}$$

Where $\mathbf{Z}_t = \mathbf{Z}'\mathbf{Y}_t$, $\mathbf{R} = \Lambda_e^{-1}\mathbf{QC}$. Decoupling the system allows checking the system stability conditions. Following Blanchard and Kahn (1980), a solution exists if the number of backward-looking variables is equal to the number of stable roots, while the number of forward-looking variables must be equal to the number of unstable roots[10]. If this condition is met, the variables will return to their long-run equilibrium path after the model has been shocked[11].

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[10]On the contrary, when the number of stable roots is greater than the number of predetermined variables there are multiple solutions. While, when the number of unstable roots is greater than the number of forward-looking variables, we have no solutions.

[11]The method of King and Watson (1998, 2002) is implemented in MATLAB: we adapt three different M-files; the first, called file *system*, contains the log-linearised model and the parameters, as calibrated in tables 3 and 4 at the end of the paper. The second, called *driver*, contains the calibration of the shocks as set in table 5. Finally, the third file calls the *system* and *driver* files and provides a routine for generating the impulse responses of the endogenous variables to the shocks. All the codes are available upon request.

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