Introduction to DSGE Models

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Introduction to DSGE Models

Program

- DSGE Introductory course (6h)
  - Object: deriving DSGE models

- Computational Macroeconomics (10h) (Prof. L. Corrado)
  - Object: techniques to solve rational expectations linear models like DSGE (requires MATLAB)

- Topics:
  - DSGE History (Galì (2008) ch.1)
  - Real business cycle models (Galì (2008) ch.2)
  - New-Keynesian models (Galì (2008) ch.3)
Motivation
Why DSGE?

- **Historical reason: Neo-Classical Synthesis**

- **Theoretical reason: Robust to Lucas (1976), Lucas and Sargent (1978) Critique**
  - Microfoundation of macroeconomic models

- **Practical reason: CBs macroeconomic models**
  - Bank of Canada (ToTEM), Bank of England (BEQM), European Central Bank (NAWM), US Federal Reserve (SIGMA), IMF (GEM), European Commission (QUEST III)
DSGE Model

What is a DSGE

- *Dynamic* means there are intertemporal problems and agents rationally form expectations;

- *Stochastic* means exogenous stochastic process may shift aggregates;

- *General Equilibrium* means that all markets are always in equilibrium:
  - Exogenous/unpredictable shocks may temporarily deviate the economy from the equilibrium.
RBC Revolution

Main Points

• Seminal papers Kydland and Prescott (1982) and Prescott (1986)

• Efficiency of the business cycle (BC)
  • BC is the outcome of the real forces in an environment with perfect competition

• Technology is the main driver of the BC
  • Technology (Total factor productivity/Solow residual) is something exogenous

• No monetary policy references
  • Including money leads to “monetary neutrality”. Money has no effects on real variables, thus CBs have no power
NK Features
Main Points

- Monopolistic Competition
  - Each firm have monopolistic power in the market she operates

- Nominal rigidities
  - Sticky price/wage

- Money is not neutral
  - Consequences of rigidities
  - However, money is neutral in the long-run
Neo-classical Synthesis

Main Points

• Use of the RBC way of modelling
  • Infinitely living agents maximize utility given by consumption and leisure
  • Firms have access to the same technology and are subjected to a random shift

• Implementation of NK Features
  • Stiky price/wage
  • Monopolistic Competition
  • Money is not neutral → CBs have room for adjusting rigidities
RBC Model

Households

Assumptions:

- Perfect competition, homogeneous goods, zero profits
- Flexible price and wage
- No capital, no investments and no government
- Discrete time
- Rationally infinity-lived price taker agents
- Complete market and perfect information
- Money is unit of account (no medium of exchange or reserve of value)
- Regularity conditions on the utility function hold
- Additively separable consumption and leisure (CRRA functional form)
  - $U$ differentiable and has continuous I, II derivatives
  - $\partial U/\partial C_t > 0$, $\partial U/\partial N_t < 0$, $\partial U/\partial C_t^2 < 0$ and $\partial U/\partial N_t^2 < 0$
RBC Model

Households

\[
\max_{C_t, N_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right)
\]

s.t.

\[
P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + T_t
\]

\[
\lim_{T \to \infty} \mathbb{E}_t \{ B_T \} \geq 0, \ \forall t
\]

**Variables:** $C_t$: consumption; $N_t$: labor; $B_t$: bond; $P_t$: price; $Q_t$: bond price; $W_t$: wage; $T_t$: lump-sum transfer/tax.

**Parameters:** $\beta$: discount factor; $\sigma$: coef. of relative risk aversion/reciprocal of intertemporal elasticity of substitution; $\phi$: inverse of the elasticity of work w.r.t. wage (inverse of Frish elasticity).
RBC Model
Households (cont’d)

F.O.C.

\[
\frac{W_t}{P_t} = N_t^\phi C_t^\sigma \tag{4}
\]

\[
\mathbb{E}_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\pi_{t+1}} \right] = Q_t \tag{5}
\]
RBC Model

Firms

\[
\max_{N_t} \quad P_t Y_t - W_t N_t
\]  

\[s.t.
\]

\[
Y_t = A_t N_t^{1-\alpha}
\]  

\[
a_t = \rho_a a_{t-1} + \epsilon_{a,t}, \quad |\rho_a| < 1, \quad \epsilon_{a,t} \sim \mathcal{N}(0, \sigma_a)
\]

Variables: \( Y_t \): output; \( A_t \): technology; \( N_t \): labor; \( P_t \): price; \( W_t \): wage; \( a_t \equiv \log(A_t) \);

Parameters: \( \alpha \) output elasticity w.r.t. labor (return to scale determinant).
RBC Model

Firms (cont’d)

\[ \frac{W_t}{P_t} = (1 - \alpha)A_t N_t^{-\alpha} \] (9)
RBC Model
Equilibrium

• Agents maximize utility subject to the budget constraint;
• Firms maximize profits subject to the production function;
• Goods and labor markets clear.

The last point in this setting without capital and government means

\[ Y_t = C_t \] (10)
**Problem**: systems of non-linear rational expectation difference equations are hard to solve.

**A possible solution**: take the log and linearize around the non-stochastic steady state using the F.O. Taylor expansion.

\[ f(x) \approx f(x_{ss}) + \frac{\partial f(x)}{\partial x}|_{x_{ss}}(x - x_{ss}) \]  

(11)
RBC Model

Log-Linearization (cont’d)

An easy way to log-linearize (up to a constant) following Uhlig (1999):

- Set \( X_t = X e^{\hat{x}_t} \) (if \( X_t^\alpha = X^\alpha e^{\alpha \hat{x}_t} \))
- Approximate \( e^{\hat{x}_t} \approx (1 + \hat{x}_t) \) (if \( e^{\alpha \hat{x}_t} \approx (1 + \alpha \hat{x}_t) \))
- \( \hat{x}_t \hat{y}_t \approx 0 \)
- Use the Steady State relationships to remove the remaining constants
RBC Model
Non-Stochastic Steady State (NSSS)

\[ Q = \beta \]  \hfill (12)

\[ \frac{W}{P} = N^\phi C^\sigma \]  \hfill (13)

\[ \frac{W}{P} = (1 - \alpha)N^{-\alpha} \]  \hfill (14)

\[ Y = N^{(1-\alpha)} \]  \hfill (15)

\[ C = Y \]  \hfill (16)
RBC Model

Log-Linear Model

\[ \hat{c}_t = \mathbb{E}_t \hat{c}_{t+1} - \sigma^{-1} \hat{r}_t \]  (17)

\[ \hat{\omega} = \phi \hat{n}_t + \sigma \hat{c}_t \]  (18)

\[ \hat{\omega} = -\alpha \hat{n}_t + a_t \]  (19)

\[ \hat{y}_t = (1 - \alpha) \hat{n}_t + a_t \]  (20)

\[ \hat{y}_t = \hat{c}_t \]  (21)
RBC Model

Log-Linear Model (cont’d)

\[ \hat{r}_t = \hat{i}_t - \mathbb{E}_t \pi_{t+1} \tag{22} \]

\[ \hat{\omega}_t = \hat{w}_t - \hat{p}_t \tag{23} \]

Results:
- Real variables are determined independently of monetary policy
- Not clear how conduct monetary policy (indeterminacy)
- Nominal variables may be pinned-down setting an interest rate rule

\[ \hat{i}_t = \phi \pi \pi_t \tag{24} \]
RBC Model
Linear Rational Expectation Model

\[ A(\Theta)^E_t x_{t+1} = B(\Theta)x_t + C(\Theta)\epsilon_t \]  \hspace{1cm} (25)

- The endogenous variables are \( x_t \equiv \{\hat{c}_t, \hat{n}_t, \hat{w}_t, \hat{y}_t, \hat{r}_t, a_t\} \).
- The exogenous variable is \( \epsilon_t \equiv \{\epsilon_{a,t}\} \).
- \( A(\Theta), B(\Theta) \) and \( C(\Theta) \) are matrices containing time invariant structural parameters.
- The parameter space is \( \Theta \equiv [\alpha, \beta, \phi, \sigma, \rho_a, \sigma_a] \).
There are many linear rational expectation solution methods:

- Balchard and Khan (1980)
- King and Watson (1998)
- Sims (2001)
- Uhlig (1999)

Returning (up to measurement errors)

\[ x_{t+1} = D(\Theta)x_t + E(\Theta)\epsilon_t \]  

Where \( D(\Theta) \) and \( E(\Theta) \) are matrices depending on parameters \( \Theta \)
Two approaches to deal with the parameters $\Theta = [\alpha, \beta, \phi, \sigma, \rho_a, \sigma_a]$

- **Calibration**
  - Calibration IS NOT estimation!
  - Long-run relationship (Hours worked per Household)
  - Results obtained in microeconomic studies (risk aversion, discount factor)

- **Estimation**
  - Matching Moments (GMM, Simulated GMM, Indirect Inference)
  - Maximum Likelihood
  - Bayesian Estimation
### RBC Model

#### Standard Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>Intertemporal elasticity of substitution</td>
<td>1.0</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Frisch elasticity of labor supply</td>
<td>1.0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Labor elasticity in the production function</td>
<td>0.36</td>
</tr>
<tr>
<td>$\phi_{\pi}$</td>
<td>Reaction coefficient on inflation</td>
<td>1.50</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Persistence of TFP shock</td>
<td>0.95</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>Volatility of TFP shock</td>
<td>0.0072</td>
</tr>
</tbody>
</table>
RBC Model

TFP shock Impulse Response Functions

- Consumption
- Nominal Interest Rate
- Inflation
- Wage
RBC Model

Simulated data

![Chart showing simulated data for RBC model.](image-url)