A Short Course on DSGE Models

Syllabus and Tentative Program

Luca Brugnolini

Central Bank of Malta – Research Department

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This short course focuses on deriving and solving a particular class of business cycle models called *Dynamic Stochastic General Equilibrium* (DSGE) models. Such models are particularly useful in assessing and interpreting the possible outcomes of policy actions – e.g., a change in the monetary policy stance – in terms of variations in economic aggregates and welfare. Also, they are used to estimate economically meaningful multipliers – e.g., the change in the national income due to a change in government spending – and to provide a historical perspective about the exogenous forces acting as drivers of the business cycle. Finally, they are capable of replicating stylized facts observed in real data, and of simulating synthetic data with a similar pattern. For these and other reasons, DSGE models are regularly employed by central banks and other institutions to test hypothetical scenarios and build confidence in policy decisions.[1]

From a theoretical point of view, the success of this class of models is linked to their capability of addressing the Lucas (1976)'s critique. Meaning, first, that the final model's equations are derived hypothesizing agents with an optimizing behavior – i.e., *microfoundation*. Secondly, it is assumed that agents base their decisions taking into account any possible future outcomes of the economy – i.e., *rational expectations* or *forward-looking behavior*.

During the course, we will see that these two assumptions create many difficulties regarding the mathematical tractability of the model. In fact, microfounding a model implies dealing

[1] For example, the Central Bank of Malta uses a small-open economy model called MEDSEA (Rapa, 2016). However, also the European Central Bank (ECB), the European Commission, the US Federal Reserve (FED), the International Monetary Fund (IMF) and many other leading institutions employ this class of models for policy analysis.

with constrained maximization problems, while the term rational expectations, considering a stochastic environment, involves dealing with intractable integrals. For these reasons, solution methods often rely on approximations and numerical – as opposed to analytical – methodologies.

During the course we will consider two simplified versions of DSGE models – i.e., the *Real Business Cycle* (RBC – Kydland and Prescott, 1982), and a minimal *New Keynesian* (NK) model. The main reference is Gali (2008). We will analytically derive the optimality and equilibrium conditions of the RBC model and compute the essential ingredients to find a solution, i.e., the *non-stochastic steady-state* and the *log-linear approximation* of the model.

In DSGE models, the equilibrium conditions always result in a **highly non-linear** system of stochastic rational expectation difference equations. This object is hard to solve directly. As a workaround, we will (log)linearize the system around a non-stochastic steady-state by applying a first-order Taylor expansion. This procedure will provide us with a **linear** system of stochastic rational expectation difference equations, giving us more chance to solve the model.

During the course, we will build the intuition of the Blanchard and Kahn (1980) and King and Watson (1998) solution methodology by analytically deriving the solution of a stylized two-equation model. Finally, we will solve a minimal NK model with the aid of MATLAB by using the toolkit described in King and Watson (2002).

References

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